

P425/1
PURE MATHEMATICS

Paper 1
Nov./Dec. 2024
3 hours



UGANDA NATIONAL EXAMINATIONS BOARD

Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

This paper consists of two Sections; A and B.

Section A is compulsory.

Answer only five questions from Section B.

Any additional question(s) answered will not be marked.

All necessary working must be shown clearly.

Begin each answer on a fresh page.

Graph paper is provided.

Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

SECTION A (40 MARKS)

Answer **all** the questions in this section.

1. A committee of seven people is to be selected from 4 men and 6 women. If the committee must have **at least** two men, determine the total possible number of ways of selecting the committee. (05 marks)
2. A cylindrical can of capacity 1000 cm^3 is made from a thin sheet of metal. The can is open at the top and closed at the bottom. The radius of the bottom is $x \text{ cm}$. Find the value of x that will minimise the area of the sheet to be used. (Leave π in your answer) (05 marks)
3. The equation of an ellipse is $4x^2 + 25y^2 + 8x - 100y + 4 = 0$.
Determine the;
(a) coordinates of the centre of the ellipse. (03 marks)
(b) eccentricity of the ellipse. (02 marks)
4. Show that $\int_0^1 \left(\frac{1}{9-x^2} \right) dx = \frac{1}{6} \ln 2$. (05 marks)
5. The population of a country increases in a geometric progression (G.P.) by 2.75 % per annum. Calculate the number of years it will take for the population to double. (05 marks)
6. Show that $\frac{1 - \cos 2x + 2 \sin x \cos^2 x}{1 + \cos 2x} = \sin x + \tan^2 x$. (05 marks)
7. The point $C(a, 4, 5)$ divides the line joining points $A(1, 2, 3)$ and $B(6, 7, 8)$ in the ratio $\lambda : 3$. Using vectors, find the values of a and λ . (05 marks)
8. Find the area enclosed by the curve $y = x^2$ and the line $y = x$ from $x = 0$ to $x = 1$. (05 marks)

SECTION B (60 MARKS)

Answer only five questions from this section.

All questions carry equal marks.

9. (a) Express $12\cos \theta + 16\sin \theta$ in the form $R \cos(\theta - \alpha)$ where R is a positive constant and α is an acute angle. (06 marks)
- (b) Hence;
- (i) find the maximum and minimum values of $12 \cos \theta + 16 \sin \theta$.
- (ii) solve the equation $12 \cos \theta + 16 \sin \theta = 15$ for $0^\circ \leq \theta \leq 180^\circ$. (06 marks)
10. (a) Given that the polynomial $x^3 - 13x + p$ is exactly divisible by $x - 4$, find the value of p . (06 marks)
- Hence solve the equation $x^3 - 13x + p = 0$.
- (b) Solve the inequality $\frac{x^2 - x - 18}{x + 3} \geq \frac{x}{2}$. (06 marks)
11. (a) Use the substitution $x = \sin \theta$ to evaluate $\int_0^1 \frac{1+x}{\sqrt{1-x^2}} dx$. (05 marks)
- (b) Given that $y = \sin^{-1} \left(\frac{x}{\sqrt{1+x^2}} \right)$, find $\frac{dy}{dx}$ in terms of x . (07 marks)
12. A curve is defined by the equations $x = -t^3 + t^2 + 1$ and $y = t^2$. A tangent to the curve at a point $B(x, y)$ is parallel to the line $3y - 2x - 1 = 0$. Determine the;
- (a) coordinates of B . (09 marks)
- (b) equation of the tangent at B . (03 marks)
13. (a) Use Maclaurin's theorem to expand $\ln(1 - 2x)$ in ascending powers of x as far as the term in x^3 . (06 marks)
- (b) Using small changes, find the approximate value of $\tan 46^\circ$ correct to three decimal places. (06 marks)

14. (a) The point C in the complex plane corresponds to the complex number z such that $3|z - 2| = |z - 6i|$. Show that the locus of C is a circle.

(05 marks)

- (b) Find the square root of $-5 + 12i$.

(07 marks)

15. The coordinates of points P and Q are $(0, 2, 5)$ and $(-1, 3, 1)$ respectively.

Given that the equation of the line T is $\frac{x-3}{2} = \frac{2-y}{2} = 2-z$;

- (a) find the equation of a plane which contains the point P and is perpendicular to the line T . (03 marks)

- (b) show that the point Q lies on the plane. (02 marks)

- (c) determine the coordinates of the point R where the line T intersects with the plane. (04 marks)

- (d) show that PR and QR are perpendicular. (03 marks)

16. The rate at which the quantity M of a commodity is sold is proportional to the difference between the amount initially present and the quantity sold at any time t . Initially 10 tonnes of the commodity were present. After one day, 2 tonnes were sold.

- (a) Form a differential equation for the quantity of the commodity sold. (02 marks)

- (b) (i) Determine the expression for M in terms of t . (08 marks)

- (ii) Calculate the quantity sold at the end of 5 days. (02 marks)



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①. 2M5W OR 3M4W OR 4M3W

$$= {}^4C_2 \times {}^6C_5 + {}^4C_3 \times {}^6C_4 + {}^4C_4 \times {}^6C_3$$

$$= (6 \times 6) + (4 \times 15) + (1 \times 20) = 36 + 60 + 20 = 116$$

Alt ${}^{10}C_7 - {}^4C_1 \times {}^6C_6 = 120 - 4 = 116$

②. $1000 = \pi x^2 h$ $h = \frac{1000}{\pi x^2}$

$$A = \pi x^2 + 2\pi x \left(\frac{1000}{\pi x^2} \right) = \pi x^2 + \frac{2000}{x}$$

$$\frac{dA}{dx} = 2\pi x - \frac{2000}{x^2}$$

for A_{min} , $\frac{dA}{dx} = 0$ $\pi x - \frac{1000}{x^2} = 0$

$$\pi x^3 = 1000$$

$$x = \frac{10}{\sqrt[3]{\pi}}$$

$$\sqrt[3]{\pi}$$



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③. $4x^2 + 8x + 25y^2 - 100y + 4 = 0$

$$4(x^2 + 2x) + 25(y^2 - 4y) + 4 = 0$$

$$4[(x+1)^2 - 1] + 25[(y-2)^2 - 4] + 4 = 0 \quad (M)$$

$$4(x+1)^2 - 4 + 25(y-2)^2 - 100 + 4 = 0$$

$$4(x+1)^2 + 25(y-2)^2 = 100$$

$$\frac{(x+1)^2}{25} + \frac{(y-2)^2}{4} = 1 \quad (A1)$$

a) Centre $(-1, 2)$ A1

b) $b^2 = a^2(1 - e^2)$ $4 = 25(1 - e^2)$

$$\frac{4}{25} = 1 - e^2 \quad (M)$$

$$e^2 = 1 - \frac{4}{25} = \frac{21}{25}$$

$$e = \frac{\sqrt{21}}{5} \quad (A1)$$

5



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$$\textcircled{4}. \frac{1}{9-x^2} \equiv \frac{A}{3+x} + \frac{B}{3-x}$$

$$A(3-x) + B(3+x) = 1$$

$$x=3 \quad 6B=1 \quad B=\frac{1}{6} \quad \textcircled{B1}$$

$$x=-3 \quad 6A=1 \quad A=\frac{1}{6}$$

$$\int_0^1 \frac{1}{9-x^2} dx = \frac{1}{6} \int_0^1 \left(\frac{1}{3+x} + \frac{1}{3-x} \right) dx$$

$$= \frac{1}{6} \left(\ln(3+x) \Big|_0^1 - \ln(3-x) \Big|_0^1 \right) \quad \textcircled{M1} \textcircled{A1}$$

$$= \frac{1}{6} (\ln 4 - \ln 3 - \ln 2 + \ln 3)$$

$$= \frac{1}{6} (2\ln 2 - \ln 2) \quad \textcircled{M1}$$

$$= \frac{1}{6} \ln 2. \quad \textcircled{B1}$$

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⑤. $U_1 = P$, $U_n = 2P$ $r = \frac{100 + 2.75}{100} = 1.0275$ (B)

$2P = P(1.0275)^{n-1}$ (m)

$2 = \frac{(1.0275)^n}{1.0275}$ $(1.0275)^n = 2 \times 1.0275 = 2.055$ (m)

$n = \log_{1.0275} 2.055 = 26.55$ (B)

$n \approx 27$ years (A)

(5)

⑥. $\frac{1 - \cos 2x + 2 \sin x \cos^2 x}{1 + \cos 2x}$

$= \frac{1 - (1 - 2 \sin^2 x) + 2 \sin x \cos^2 x}{1 + 2 \cos^2 x - 1}$ (m) (m)

$= \frac{1 - 1 + 2 \sin^2 x + 2 \sin x \cos^2 x}{2 \cos^2 x}$ (m)

$= \frac{2 \sin x \cos^2 x}{2 \cos^2 x} + \frac{2 \sin^2 x}{2 \cos^2 x}$ (m)

$= \sin x + \tan^2 x$ (B)

(B)
 $\frac{2 \sin^2 x + 2 \sin x \cos^2 x}{2 \cos^2 x \times B1}$
 $\frac{2 \sin^2 x}{2 \cos^2 x} + \frac{2 \sin x \cos^2 x}{2 \cos^2 x} \times \frac{m}{m}$
 $\tan^2 x + \sin x \times A1$

(5)



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7.
$$C = \frac{3a + \lambda b}{3 + \lambda}$$

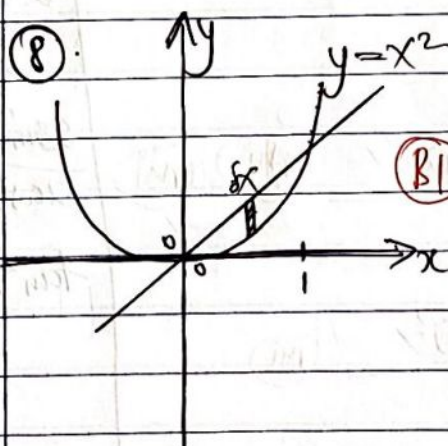
$$(3 + \lambda) \begin{pmatrix} 9 \\ 4 \\ 5 \end{pmatrix} = 3 \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 6 \\ 7 \\ 8 \end{pmatrix} = \begin{pmatrix} 3 + 6\lambda \\ 6 + 7\lambda \\ 9 + 8\lambda \end{pmatrix} \quad (M) \quad (M)$$

$$12 + 4\lambda = 6 + 7\lambda \quad A/B \quad 15 + 5\lambda = 9 + 8\lambda \quad (M)$$

$$6 = 3\lambda \quad 3\lambda = 6$$

$$\lambda = 2 \quad (A) \quad \lambda = 2$$

$$(3 + 2)a = 3 + 6(2) \Rightarrow 5a = 15 \quad a = 3 \quad (A) \quad (5)$$



$$x^2 = x$$

$$x(x - 1) = 0 \quad x = 0, 1$$

$$A = \int_0^1 (x - x^2) dx \quad (B) \quad (M)$$

$$= \left(\frac{x^2}{2} - \frac{x^3}{3} \right) \Big|_0^1 \quad (M)$$

$$= \frac{1}{2} - \frac{1}{3} \quad (M)$$

$$= \frac{1}{6} \text{ sq. units} \quad (A) \quad (5)$$



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9. a) $12 \cos \theta + 16 \sin \theta = R \cos(\theta - \alpha)$

$12 \cos \theta + 16 \sin \theta = R \cos \theta \cos \alpha + R \sin \theta \sin \alpha$ (M1)

$R = \sqrt{12^2 + 16^2} = 20$ (M1) (A1), $R \cos \alpha = 12$, $\cos \alpha = \frac{12}{20}$ (M1)

$\alpha = 53.13^\circ$ (A1)

$12 \cos \theta + 16 \sin \theta = 20 \cos(\theta - 53.13^\circ)$ (B1)

b) (i) Max value = 20 (A1), Min value = -20 (A1)

(ii) $12 \cos \theta + 16 \sin \theta = 15$

$20 \cos(\theta - 53.13^\circ) = 15$

$\cos(\theta - 53.13^\circ) = \frac{15}{20} = \frac{3}{4}$ (B1)

$\theta - 53.13^\circ = \pm 41.41^\circ$ (M1)

$\theta = 11.72^\circ, 94.54^\circ$ (A1) (A1)

(12)



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10. a) $4^3 - 12(4) + p = 0$ (M1)
 $64 - 52 + p = 0$
 $p = -12$ (A1)

$$\begin{array}{r} x^2 + 4x + 3 \\ x-4 \overline{) x^3 - 13x - 12} \quad \text{(M1)} \\ \underline{-(x^3 - 4x^2)} \\ 4x^2 - 13x - 12 \\ \underline{-(4x^2 - 16x)} \\ 3x - 12 \\ \underline{-(3x - 12)} \\ 0 \end{array}$$

$$\begin{array}{r} x^3 \quad x^2 \quad x \quad x^0 \\ x=4 \quad \begin{array}{|c|c|c|c|} \hline 1 & 0 & -13 & -12 \\ \hline 0 & 4 & 16 & 12 \\ \hline 1 & 4 & 3 & 0 \\ \hline \end{array} \\ \\ x=-1 \quad \begin{array}{r} x^2 \quad x \quad x \\ \hline 1 \quad 4 \quad 3 \\ 0 \quad -1 \quad -3 \\ \hline 1 \quad 3 \quad 0 \end{array} \end{array}$$

$(x-4)(x^2 + 4x + 3) = 0$

$(x-4)(x+3)(x+1) = 0$ (M1)

$x = 4, -3, -1$ (A1) (A1)

b) $\frac{x^2 - x - 18}{x+3} \geq \frac{x}{2}$

$\frac{x^2 - x - 18}{x+3} - \frac{x}{2} \geq 0$

$\frac{2x^2 - 2x - 36 - x^2 - 3x}{2(x+3)} \geq 0$ (M1)

$2(x+3)$



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$$\frac{x^2 - 5x - 36}{2(x+3)} \geq 0$$

$$\frac{(x-9)(x+4)}{2(x+3)} \geq 0 \quad (M1)$$

Critical values $x = -4, -3, 9$ (B1) for all

	$x < -4$	$-4 < x < -3$	$-3 < x < 9$	$x > 9$	
$\frac{x^2 - 5x - 36}{2(x+3)}$	-	+	-	+	(M1)

$$\underline{-4 \leq x < -3}, \quad \underline{x \geq 9}$$

(A1)

(A1)

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11. a) $X = \sin \theta$ $dx = \cos \theta d\theta$ $\begin{matrix} X & 0 & 1 \\ \theta & 0 & \pi/2 \end{matrix}$

$$\int_0^1 \frac{1+x}{\sqrt{1-x^2}} dx = \int_0^{\pi/2} \frac{1+\sin \theta}{\sqrt{1-\sin^2 \theta}} \cdot \cos \theta d\theta = \int_0^{\pi/2} (1+\sin \theta) d\theta$$

$$= \theta \Big|_0^{\pi/2} - \cos \theta \Big|_0^{\pi/2}$$

$$= \frac{\pi}{2} - 0 - \left(\cos \frac{\pi}{2} - \cos 0 \right)$$

$$= \frac{\pi}{2} + 1$$

b) $y = \sin^{-1} \left(\frac{x}{(1+x^2)^{1/2}} \right)$ $\sin y = \frac{x}{(1+x^2)^{1/2}}$

$$\cos y \frac{dy}{dx} = \frac{(1+x^2)^{1/2} - \frac{(2x)(x)}{2(1+x^2)^{1/2}}}{1+x^2}$$

$$\sqrt{1-\frac{x^2}{1+x^2}} \frac{dy}{dx} = \frac{1+x^2-x^2}{(1+x^2)^{3/2}}$$

$$\frac{\sqrt{1+x^2-x^2}}{\sqrt{1+x^2}} \frac{dy}{dx} = \frac{1}{(1+x^2)^{3/2}}$$

$$\frac{1}{(1+x^2)^{1/2}} \frac{dy}{dx} = \frac{1}{(1+x^2)^{3/2}}$$

$$\frac{dy}{dx} = \frac{1}{1+x^2}$$



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(12) a) $x = -t^3 + t^2 + 1$, $y = t^2$
 $\frac{dx}{dt} = -3t^2 + 2t$ (mi), $\frac{dy}{dt} = 2t$ (mi)

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \frac{2t}{-3t^2 + 2t}$ (mi) = $\frac{2}{2-3t}$

$3y - 2x - 1 = 0$, $y = \frac{2}{3}x + \frac{1}{3}$

$\frac{2}{3} = \frac{2t}{-3t^2 + 2t}$ (mi) $-3t^2 + 2t = 3t$

$3t^2 + t = 0$ $t(3t+1) = 0$ (mi)
 $t = 0, -\frac{1}{3}$ $\therefore t = -\frac{1}{3}$ (A1)

$x = -(-\frac{1}{3})^3 + (-\frac{1}{3})^2 + 1 = 1 + \frac{1}{9} + \frac{1}{9} = \frac{31}{27}$ (mi) $\frac{2}{2-3t} = \frac{2}{3}$

$y = (-\frac{1}{3})^2 = \frac{1}{9}$ (mi)

$B(\frac{31}{27}, \frac{1}{9})$ (A1)

b) $3y - 2x + k = 0$

$3(\frac{1}{9}) - 2(\frac{31}{27}) + k = 0$ (mi) $k = \frac{62}{27} - \frac{1}{3} = \frac{53}{27}$ (A1)

$3y - 2x + \frac{53}{27} = 0$

$81y - 54x + 53 = 0$ (A1)



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13. a) $f(x) = \ln(1-2x)$, $f(0) = \ln 1 = 0$

$f'(x) = \frac{-2}{1-2x}$ (B1) $f'(0) = \frac{-2}{1} = -2$

$f''(x) = \frac{-4}{(1-2x)^2}$ (B1) $f''(0) = \frac{-4}{1} = -4$

$f'''(x) = \frac{-16}{(1-2x)^3}$ (B1) $f'''(0) = \frac{-16}{1} = -16$

$\ln(1-2x) = -2x - \frac{4x^2}{2!} - \frac{16x^3}{3!}$ (M1) (M1)

$= -2x - 2x^2 - \frac{8x^3}{3}$ (A1)

b) $y = \tan x$, $y + \delta y = \tan(x + \delta x)$

$\tan 46^\circ = \tan(45^\circ + 1^\circ)$ $x = 45^\circ = \frac{\pi}{4}$, $\delta x = 1^\circ = \frac{\pi}{180}$ (B1)

$\frac{dy}{dx} = \sec^2 x = 1 + \tan^2 x$ (M1)

$\frac{\delta y}{\delta x} \approx \frac{dy}{dx}$ $y = \tan \frac{\pi}{4} = 1$, $\delta y = \left(1 + \tan^2 \frac{\pi}{4}\right) \cdot \frac{\pi}{180}$ (M1) (A1)

$\tan 46^\circ = 1 + \frac{\pi}{90} \approx 1.035$ (B1) (12)



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14. a) $z = x + yi$

$$3|x + yi - 2| = |x + yi - 6i|$$

$$3|x - 2 + yi| = |x + (y - 6)i|$$

$$3\sqrt{(x-2)^2 + y^2} = \sqrt{x^2 + (y-6)^2} \quad (m)$$

$$9(x^2 - 4x + 4 + y^2) = x^2 + y^2 - 12y + 36 \quad (m) \quad (m)$$

$$9x^2 - 36x + 36 + 9y^2 = x^2 + y^2 - 12y + 36$$

$$8x^2 + 8y^2 - 36x + 12y = 0 \quad (m)$$

$$2x^2 + 2y^2 - 9x + 3y = 0 \quad (B)$$

b) $\sqrt{-5 + 12i} = \sqrt{-5 + 2(6i)} = \sqrt{-5 + 2(2)(3i)} \quad (m)$

$$= \sqrt{2^2 + 2(2)(3i) + (3i)^2} \quad (m) \quad (m) \quad (m)$$

$$= \sqrt{(2 + 3i)^2} \quad (m)$$

$$\sqrt{-5 + 12i} = \pm (2 + 3i) \quad (A) \quad (A)$$

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Alt $|-5+12i| = \sqrt{(-5)^2+12^2} = 13$

$$\arg(-5+12i) = 180^\circ - \tan^{-1}\left(\frac{12}{5}\right) = 112.62^\circ$$

$$-5+12i = 13 (\cos 112.62^\circ + i \sin 112.62^\circ)$$

$$\sqrt{-5+12i} = \sqrt{13} \left\{ \cos \left(\frac{112.62^\circ + 360^\circ k}{2} \right) + i \sin \left(\frac{112.62^\circ + 360^\circ k}{2} \right) \right\}$$

$$k=0 \quad \sqrt{13} (\cos 56.31^\circ + i \sin 56.31^\circ) = \underline{\underline{2+3i}}$$

$$k=1 \quad \sqrt{13} (\cos 236.31^\circ + i \sin 236.31^\circ) = \underline{\underline{-2-3i}}$$

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15. P(0, 2, 5) Q(-1, 3, 1) T $\frac{x-3}{2} = \frac{y-2}{-2} = \frac{z-2}{-1}$

$$\vec{r} = \begin{pmatrix} 3 \\ 2 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -2 \\ -1 \end{pmatrix}$$

a) $\begin{pmatrix} x \\ y-2 \\ z-5 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -2 \\ -1 \end{pmatrix} = 0$ (M1) $2x - 2y + 4 - z + 5 = 0$ (M1)
 $2x - 2y - z = -9$ (A1)

b) $2(-1) - 2(3) - 1 = -2 - 6 - 1 = -9$ (M1) (B1)

c) $x = 3 + 2\lambda$, $y = 2 - 2\lambda$, $z = 2 - \lambda$

$$2(3 + 2\lambda) - 2(2 - 2\lambda) - (2 - \lambda) = -9$$
 (M1)
 $6 + 4\lambda - 4 + 4\lambda - 2 + \lambda = -9$
 $9\lambda = -9$

$$\lambda = -1$$
 (A1)

$$x = 3 + 2(-1) = 1, y = 2 - 2(-1) = 4, z = 2 - (-1) = 3$$
 (M1)

$$R(1, 4, 3)$$
 (A1)

d) $\vec{PR} = \begin{pmatrix} 1 \\ 4 \\ 3 \end{pmatrix} - \begin{pmatrix} 0 \\ 2 \\ 5 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$ (B1) $\vec{QR} = \begin{pmatrix} 1 \\ 4 \\ 3 \end{pmatrix} - \begin{pmatrix} -1 \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$ (B1)

$$\vec{PR} \cdot \vec{QR} = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} = 2 + 2 - 4 = 0$$
 (B1)



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(16) a) $\frac{dM}{dt} \propto (10-M)$, $\frac{dM}{dt} = k(10-M)$ (m1) (A1)

b) $\int_0^M \frac{dM}{10-M} = \int_0^t k dt$ (m1)

$-\ln(10-M) \Big|_0^M = kt \Big|_0^t$ (m1) (m1) (A1)

$-\left[\ln(10-M) - \ln 10\right] = k(t-0)$ (m1)

$-\ln(10-M) + \ln 10 = kt$

At $t=1$, $M=2$ $-\ln 8 + \ln 10 = k$ (m1) $k = \ln\left(\frac{10}{8}\right) = \ln\left(\frac{5}{4}\right)$ (A1)

$-\ln(10-M) + \ln 10 = t \ln\left(\frac{5}{4}\right)$

$\ln(10-M) = \ln 10 - t \ln\left(\frac{5}{4}\right) = \ln 10 + t \ln\left(\frac{4}{5}\right)$

$\ln(10-M) = \ln 10 \left(\frac{4}{5}\right)^t$ (m1)

$10-M = 10 \left(\frac{4}{5}\right)^t$ $M = 10 - 10 \left(\frac{4}{5}\right)^t$ (A1)

(ii) $M(5) = 10 - 10 \left(\frac{4}{5}\right)^5$ (m1)

$M = 6.7232 \text{ tonnes}$ (A1)

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Uganda Advanced Certificate of Education

APPLIED MATHEMATICS

Paper 2

3 hours

INSTRUCTIONS TO CANDIDATES:

This paper consists of two Sections; A and B.

Section A is compulsory.

Answer only five questions from Section B.

Any additional question(s) answered will not be marked.

All necessary working must be shown clearly.

Begin each answer on a fresh page.

Graph paper is provided.

Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

In numerical work, take acceleration due to gravity g , to be 9.8 ms^{-2} .

SECTION A (40 MARKS)

Answer all the questions in this section.

1. Two events A and B are such that $P(A) = 0.4$, $P(B) = 0.7$ and $P(A \cap B) = 0.35$. Find $P(A' \cap B')$. (05 marks)
2. The resultant of two forces P and Q is a force of 15 N acting at an angle of 50° to force Q . Given that the magnitude of force Q is 12 N, determine the magnitude and direction of the force P . (05 marks)
3. The table below shows the velocity V at any instant t seconds of a particle moving in a straight line.

t (s)	0	1	2	3	4
V (ms ⁻¹)	0	2	7	8	10

Calculate;

- (a) the velocity of the particle after 1.5 seconds. (03 marks)
- (b) the time when the velocity is 13 ms⁻¹. (02 marks)
4. The table below shows the unit prices (Shs) and quantities of food items in the years 2005 and 2010.

FOOD ITEM	QUANTITY	PRICE (SHS)	
		2005	2010
Sugar	25 kg	2500	3500
Meat	10 kg	5000	7000
Beans	50 kg	1500	2000
Fish	5 pieces	5000	8000
Maize flour	50 kg	800	1200

- (a) Calculate the weighted price index of 2010 using 2005 as the base year. (04 marks)
- (b) Comment on your result. (01 mark)
5. A force F (N) is acting on a particle of mass 4 kg whose position vector at any time t is $\mathbf{r} = (t^3 \mathbf{i} + \sin t \mathbf{j})$ m. Find F when $t = \frac{\pi}{3}$ (s). (05 marks)

6. A metallic container is in form of a cuboid. Its dimensions are 2.7 m, 4.80 m and 3.281 m correct to the given number of decimal places. Determine the possible minimum and maximum volumes of the container correct to **three** decimal places. (05 marks)

7. A discrete random variable X , has a probability function given by

$$P(X=x) = \begin{cases} \frac{1}{10}x; & 1, 2, \dots, n \\ 0, & \text{elsewhere.} \end{cases}$$

Given that $E(X) = 3$, find the value of n . (05 marks)

8. A boy can swim in still water at a speed of 2.5 ms^{-1} . The boy wishes to cross a straight river which is 50 m wide and flowing at a speed of 3 ms^{-1} . He sets off at an angle of 60° to the bank of the river.

Determine;

- (a) the time it takes him to cross the river. (03 marks)
(b) the boy's resultant velocity. (02 marks)

SECTION B (60 MARKS)

Answer any **five** questions from this section.

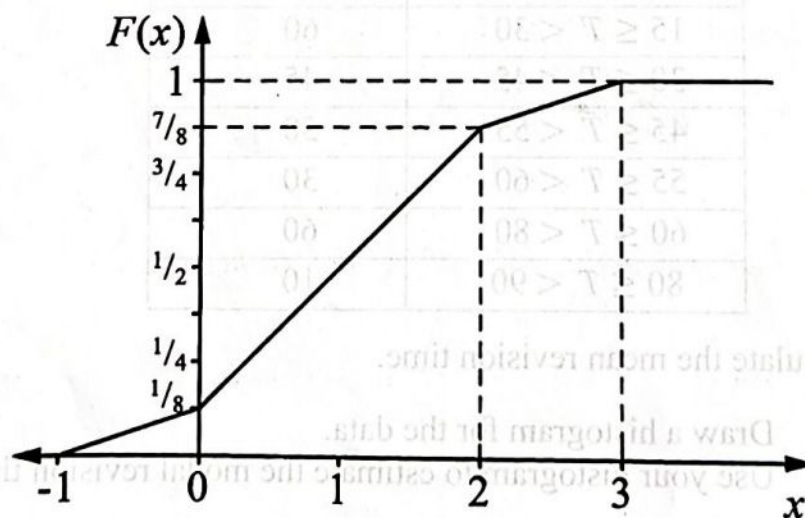
All questions carry equal marks.

9. The table below shows the distribution of time (minutes) spent by students revising for a test.

Time, T (minutes)	Frequency
$0 \leq T < 10$	20
$10 \leq T < 15$	18
$15 \leq T < 30$	60
$30 \leq T < 45$	45
$45 \leq T < 55$	50
$55 \leq T < 60$	30
$60 \leq T < 80$	60
$80 \leq T < 90$	10

- (a) Calculate the mean revision time. (05 marks)
(b) (i) Draw a histogram for the data.
(ii) Use your histogram to estimate the modal revision time. (07 marks)

10. (a) A ball was thrown with a velocity of 15 ms^{-1} vertically upwards from the ground. A girl on a balcony of a building 9 metres high, leaned and caught the ball on its way down.
- Calculate the time taken before the ball is caught by the girl. (04 marks)
 - Find the speed with which the ball was travelling when it was caught. (03 marks)
- (b) A particle travels with an initial velocity of $(11\mathbf{i} - 8\mathbf{j} + 3\mathbf{k}) \text{ ms}^{-1}$ from a point with a position vector of $(-2\mathbf{i} + \mathbf{j}) \text{ m}$. The particle moves with an acceleration of $\frac{1}{5}(2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}) \text{ ms}^{-2}$. Determine;
- the position vector of the particle after 5 seconds. (03 marks)
 - the distance covered in the 5 seconds. (02 marks)
11. (a) Using the trapezium rule with six ordinates, estimate $\int_0^1 xe^{(x^2+1)} dx$ giving your answer correct to **three** decimal places. (07 marks)
- (b) Given that $\int_0^1 xe^{(x^2+1)} dx = 2.335$, find the percentage error for the estimate in (a). (03 marks)
- (c) Suggest how the accuracy in (a) can be improved. (02 marks)
12. The cumulative distribution function $F(x)$ of a continuous random variable X is represented graphically as shown below.



Find:

- (a) (i) $F(x)$. (05 marks)
(ii) $P(1 < X < 2.5)$. (02 marks)

- (b) (i) the probability density function (pdf) $f(x)$. (02 marks)
(ii) $E(X)$. (03 marks)

13. A light inelastic string of length 80 cm is fixed at one end R and carries a particle of mass 0.1 kg at the other end S . The particle moves in a horizontal circle with angular speed 5 rad s^{-1} . Determine;

(a) the tension in the string. (08 marks)

(b) the radius of the horizontal circle. (04 marks)

14. (a) Use the graphical method to estimate the root of the equation $2x^3 - 4x + 3 = 0$ in the interval $-2 \leq x \leq -1$. (06 marks)

(b) Using the Newton Raphson method, find the root of the equation $2x^3 - 4x + 3 = 0$ taking the approximate root obtained in (a) as the initial value of x_0 . Give your answer correct to **three** decimal places. (06 marks)

15. An examination consists of 120 multiple choice questions. Each question has four options for which there is only one correct option.

(a) If a candidate who sat for the examination is chosen at random, find the probability that the candidate obtained;

(i) between 20 and 40 (inclusive) correct options.

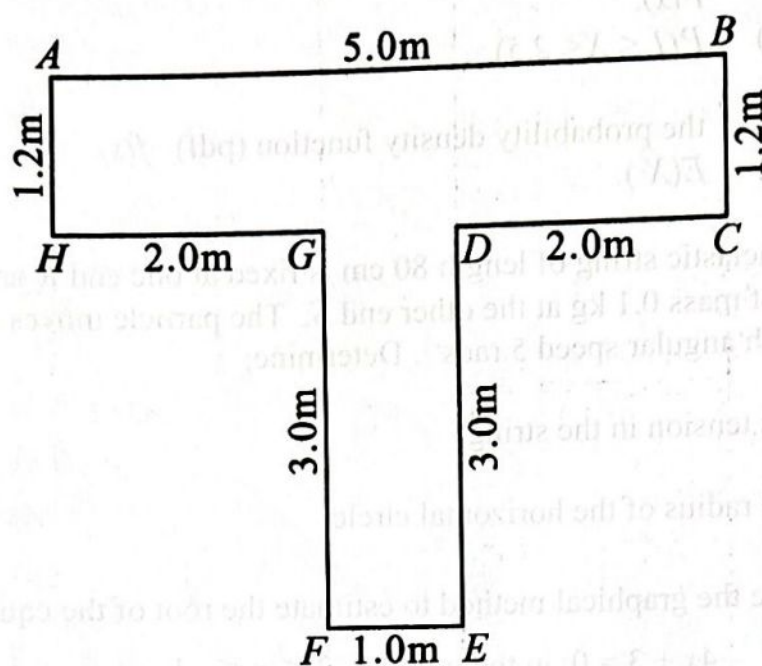
(ii) exactly 41 correct options.

(07 marks)

(b) Determine the pass mark for 80 % of the candidates to pass the examination.

(05 marks)

16. The diagram below shows a uniform lamina $ABCDEFGH$.



- (a) Determine the distances of the centre of gravity of the lamina from the sides AB and AH . (09 marks)
- (b) If the lamina is suspended from the vertex A and rests in equilibrium, calculate the angle which the side AB makes with the vertical. (03 marks)



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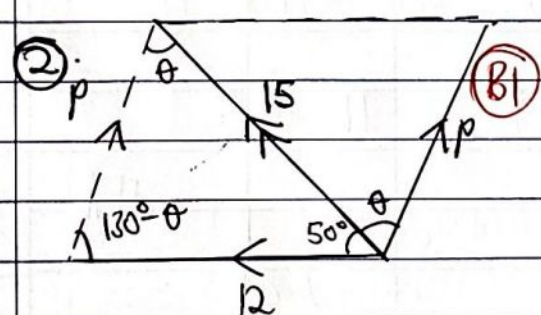
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2024 UACE P425/2

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①. $P(A \cup B) = 0.4 + 0.7 - 0.35 = 0.75$

$P(A'/B') = \frac{P(A' \cap B')}{P(B')} = \frac{1 - 0.75}{1 - 0.7} = \frac{0.25}{0.3} = \frac{5}{6}$ OR 0.8333



$P^2 = 12^2 + 15^2 - 2(12)(15)\cos 50^\circ$

$P = 11.7302 \text{ N}$

$\frac{11.7302}{\sin 50^\circ} = \frac{12}{\sin \theta}$

$\theta = \sin^{-1} \left(\frac{12 \sin 50^\circ}{11.7302} \right) = 51.6^\circ$

11.7302 N at 101.6° with 12 N

③. a)

1	1.5	2
2	V	7

$\frac{V-2}{1.5-1} = \frac{7-2}{2-1}$

$V = \frac{9}{2}$ or 4.5 m s⁻¹

b)

3	4	t
8	10	13

$\frac{t-4}{13-10} = \frac{4-3}{10-8}$

$t = \frac{11}{2}$ or 5.5 s



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④. a) XAPI

$$= \frac{(3500 \times 25) + (7000 \times 10) + (2000 \times 50) + (8000 \times 5) + (1200 \times 50)}{(2500 \times 25) + (5000 \times 10) + (1500 \times 50) + (5000 \times 5) + (800 \times 50)} \times 100$$

$$= \frac{357,500}{252,500} = 1.4158 \text{ OR } 141.58 \approx \underline{142} \quad \text{(M1) (M1) (M1) (A1)}$$

b) increase of 42% (R1)

Alt a) AKPI

$$= 25 \left(\frac{3500}{2500} \times 100 \right) + 10 \left(\frac{7000}{5000} \times 100 \right) + \left(\frac{2000}{1500} \times 100 \right) 50 + 5 \left(\frac{8000}{5000} \times 100 \right) + 50 \left(\frac{1200}{800} \times 100 \right)$$

$$= 25 + 10 + 50 + 5 + 50 \quad \text{(M1) (M1) (M1)}$$

$$= 3500 + 1400 + 20000 + 800 + 7500 = 141.90 \approx \underline{142} \quad \text{(A1)}$$

140

⑤. $\underline{r} = \begin{pmatrix} t^3 \\ \sin t \end{pmatrix}_m \quad \frac{d\underline{r}}{dt} = \begin{pmatrix} 3t^2 \\ \cos t \end{pmatrix}_{m-1} \quad \frac{d^2 \underline{r}}{dt^2} = \begin{pmatrix} 6t \\ -\sin t \end{pmatrix}_{m-2}$ (R1) (R1)

$\underline{F} = 4 \begin{pmatrix} 6t \\ -\sin t \end{pmatrix}_N \quad \underline{F}\left(\frac{\pi}{3}\right) = 4 \begin{pmatrix} 6(\frac{\pi}{3}) \\ -\sin \frac{\pi}{3} \end{pmatrix}_N = \begin{pmatrix} 8\pi \\ -2\sqrt{3} \end{pmatrix}_N$ (M1) (M1) (A1)

OR $\underline{F} = (8\pi \underline{i} - 2\sqrt{3} \underline{j}) N$



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⑥. $V_{max} = 2.75 \times 4.805 \times 3.215 = 43.361 \text{ m}^3$
 (B1) (M) (A1)

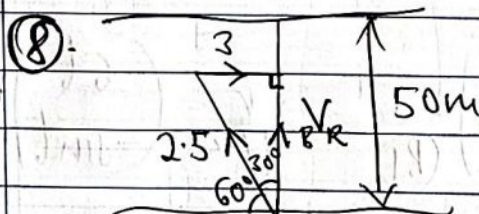
$V_{min} = 2.65 \times 4.795 \times 3.205 = 41.684 \text{ m}^3$
 (M) (A1)

⑦. $\frac{1}{10} (1+2+\dots+n) = 1$
 (M)

$$\frac{n(n+1)}{2} = 10$$

$n^2 + n - 20 = 0$ (B1) $(n-4)(n+5) = 0$ (M)
 $n = 4, -5$ (B1)
 $\therefore \underline{n = 4}$ (A1)

~~A1~~ $\frac{1}{10} (1^2 + 2^2 + \dots + n^2) = 3$ $\frac{n(n+1)(2n+1)}{6} = 30$
 $\frac{20}{6} (2n+1) = 30$
 $2n+1 = 9$
 $\underline{n = 4}$



a) $f = \frac{50}{2.5 \cos 30^\circ} = 23.09 \text{ s}$ (M) (A1)

b) $R V_R = 2.5 \cos 30^\circ = 2.1651 \text{ m s}^{-1}$ (M)



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9.

c-l	x	f	xf	i	f.i = f/i
0-10	5	20	100	10	2
10-15	12.5	18	225	5	3.6
15-30	22.5	60	1350	15	4
30-45	37.5	45	1687.5	15	3
45-55	50	50	2500	10	5
55-60	57.5	30	1725	5	6
60-80	70	60	4200	20	3
80-90	85	10	850	10	1

$\Sigma f = 293$, $\Sigma xf = 12637.5$

a) Mean = $\frac{12637.5}{293} = 43.1314$

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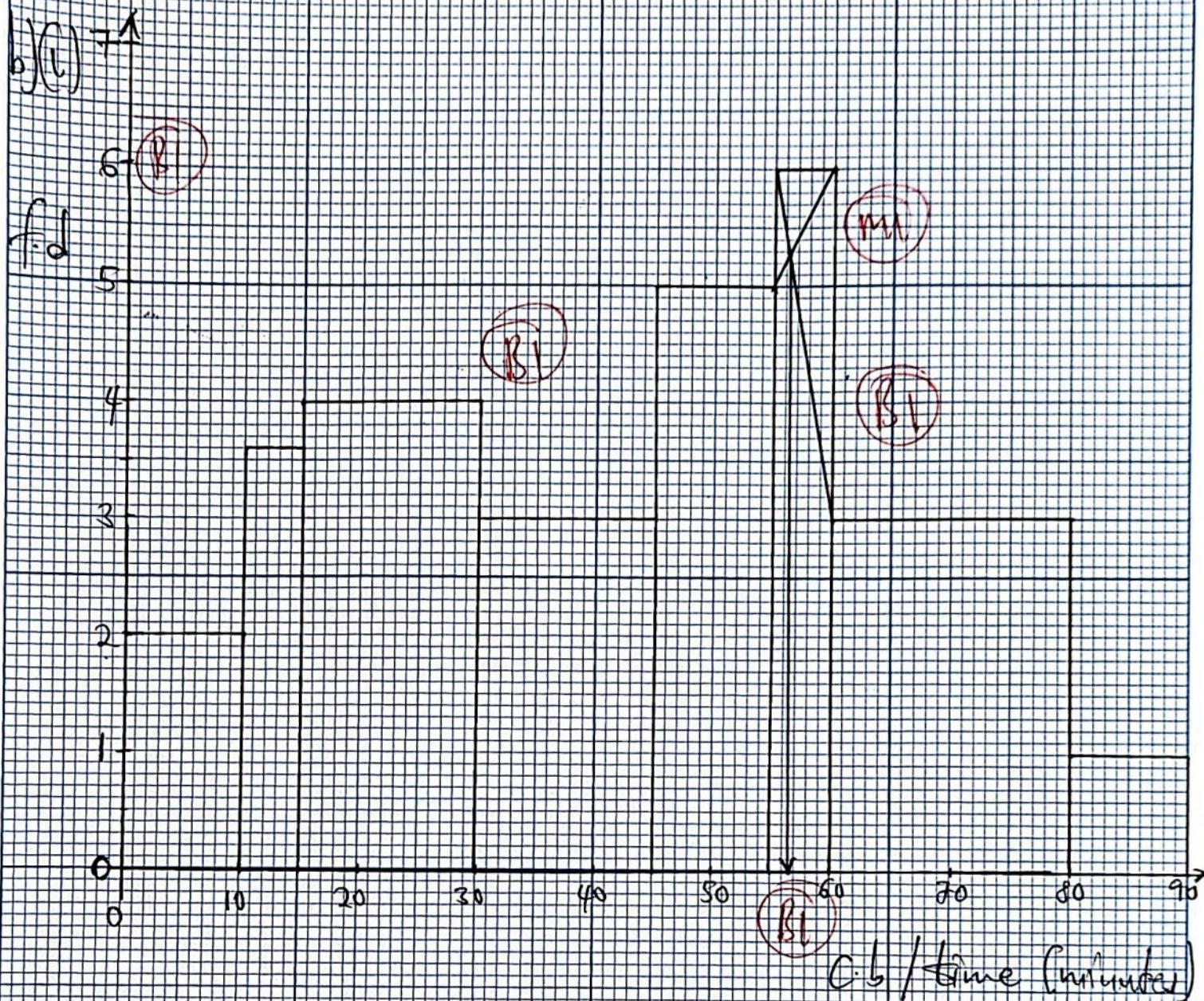


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(ii) Mode = 56.5 ± 0.5 minutes (A1)

12



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10. a) (i) $H = \frac{u^2}{2g} = \frac{15^2}{2(9.8)} = \frac{1125}{98} \text{ m} = 11.48 \text{ m}$ (B1)

$h = \frac{1}{2}gt^2$ $11.48 - 9 = \frac{1}{2}(9.8)t^2$ (M1) (M2)
 $t = 0.7114 \text{ s}$ (A1)

(ii) $V = gt = 9.8 \times 0.7114 = 6.9717 \text{ ms}^{-1}$ (M1) (M2) (A1)

b) (i) $\underline{r} = \underline{ut} + \frac{1}{2}\underline{at}^2$

$\underline{r} = 5 \begin{pmatrix} 11 \\ -9 \\ 3 \end{pmatrix} + \frac{1}{2} \times 5^2 \begin{pmatrix} 0.4 \\ 0.6 \\ -0.8 \end{pmatrix} = \begin{pmatrix} 55 \\ -45 \\ 15 \end{pmatrix} + \begin{pmatrix} 5 \\ 7.5 \\ -10 \end{pmatrix}$ (M1)

$\underline{r} = \begin{pmatrix} 60 \\ -32.5 \\ 5 \end{pmatrix} \text{ m}$ (A1)

(ii) $\underline{r} = \begin{pmatrix} 60 \\ -32.5 \\ 5 \end{pmatrix} - \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 62 \\ -33.5 \\ 5 \end{pmatrix}$ (B1)

$D = \sqrt{62^2 + (-33.5)^2 + 5^2} = 70.65 \text{ m}$ (M1) (A1)

(12)



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11. a) $h = \frac{1-0}{6-1} = \frac{1}{5} = 0.2$ $y = xe^{(x^2+1)}$

x	0	0.2	0.4	0.6	0.8	1	Sum
y	0	0.5658	1.2760	2.3377	4.1241	7.3891	7.3891

$$\int_0^1 xe^{(x^2+1)} dx = \frac{1}{2} \times 0.2 (7.3891 + 2 \times 8.3036)$$

$$= 2.39963 \approx 2.400$$

b) P.E = $\left| \frac{2.335 - 2.400}{2.335} \right| \times 100$

$$= 2.784\%$$

c) Increasing on number of ordinates.

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②. a) (1) $-1 < x < 0$ $y = mx + c$
 $(-1, 0)$ $(0, \frac{1}{8})$ $\frac{1}{8} = c$

$x < -1$

$F(x) = 0$

$F(x) = \frac{1}{8}x + \frac{1}{8}$ (B1)

$0 = -m + c$

$m = c$

$0 < x < 2$

$(0, \frac{1}{8}), (2, \frac{7}{8})$

$\frac{1}{8} = c$

$\frac{7}{8} = 2m + \frac{1}{8}$

$2m = \frac{3}{4}$

$m = \frac{3}{8}$

$F(x) = \frac{3}{8}x + \frac{1}{8}$ (B1)

$2 < x < 3$

$1 = 3m + c$

$1 = 3(\frac{1}{8}) + c$

$(2, \frac{7}{8}), (3, 1)$

$\frac{7}{8} = 2m + c$

$c = \frac{5}{8}$

$\frac{1}{8} = m$

$F(x) = \frac{1}{8}x + \frac{5}{8}$ (B1)

$x \geq 3$

$F(x) = 1$

$F(x) = \begin{cases} 0 & x < -1 \\ \frac{1}{8}x + \frac{1}{8} & -1 < x < 0 \\ \frac{3}{8}x + \frac{1}{8} & 0 < x < 2 \\ \frac{1}{8}x + \frac{5}{8} & 2 < x < 3 \\ 1 & x \geq 3 \end{cases}$ (M1) (A1)



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$$(i) P(1 < X < 2.5) = F(2.5) - F(1)$$

$$= \left(\frac{1}{8} \left(\frac{5}{2} \right) + \frac{5}{8} \right) - \left(\frac{3}{8} (1) + \frac{1}{8} \right) = \frac{5}{16} + \frac{5}{8} - \frac{1}{2}$$

$$= \frac{7}{16}$$

b) (i)

$$x < -1$$

$$-1 < x < 0$$

$$0 < x < 2$$

$$2 < x < 3$$

$$F(x) = 0$$

$$F(x) = \frac{1}{8}x + \frac{1}{8}$$

$$F(x) = \frac{3}{8}x + \frac{1}{8}$$

$$F(x) = \frac{1}{8}x + \frac{5}{8}$$

$$F'(x) = 0$$

$$F'(x) = \frac{1}{8}$$

$$F'(x) = \frac{3}{8}$$

$$F'(x) = \frac{1}{8}$$

$$x \geq 3$$

$$F(x) = 1, F'(x) = 0$$

for all correct derivatives

$$f(x) = \begin{cases} \frac{1}{8} & -1 < x < 0 \\ \frac{3}{8} & 0 < x < 2 \\ \frac{1}{8} & 2 < x < 3 \\ 0 & \text{elsewhere} \end{cases}$$

$$(ii) E(X) = \int_{-1}^0 \frac{1}{8}x dx + \int_0^2 \frac{3}{8}x dx + \int_2^3 \frac{1}{8}x dx$$

$$= \frac{x^2}{16} \Big|_{-1}^0 + \frac{3x^2}{16} \Big|_0^2 + \frac{x^2}{16} \Big|_2^3 = \frac{1}{16} (0-1) + \frac{3}{16} (4-0) + \frac{1}{16} (9-4)$$

$$= \frac{3}{4} - \frac{1}{16} + \frac{5}{16} = 1$$



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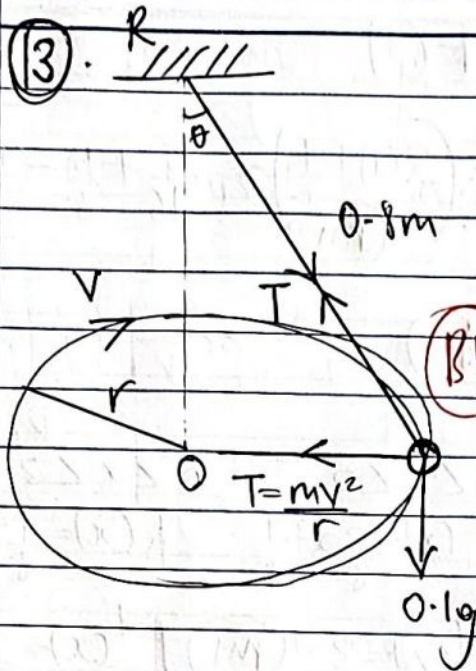
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$$\omega = 5 \text{ rad s}^{-1}$$

$$\omega = \frac{v}{r} \Rightarrow v = r\omega$$

$$T = \frac{m(r\omega)^2}{r} = mr\omega^2$$

$$\rightarrow T \sin \theta = mr\omega^2 = 0.1 \times 5^2 r = 2.5r \quad \text{--- (M1) (B1)}$$

$$\uparrow T \cos \theta = 0.1g = 0.1 \times 9.8 = 0.98 \quad \text{--- (M1) (B1)}$$

$$\frac{\sin \theta}{\cos \theta} = \frac{r}{0.8} \quad r = 0.8 \tan \theta \quad \text{(B1)}$$

a) $\frac{Tr}{0.8} = 2.5r \quad T = 0.8 \times 2.5 = 2 \text{ N} \quad \text{(M1) (A1)}$

b) $2 \cos \theta = 0.98$
 $\cos \theta = 0.49 \quad \text{(B1)} \quad \sin \theta = \sqrt{1 - (0.49)^2} = 0.8717 \quad \text{(B1)}$
 $r = 0.8 \times 0.8717 = 0.69736 \text{ m} \quad \text{(M1) (A1)}$



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(14). a) $y = 2x^2 - 4x + 3$

x	-2	-1.8	-1.6	-1.4	-1.2	-1	(B1)
y	-5	-1.5	1.2	3.1	4.3	5	(B1)

b) $x_1 = -1.7 - \frac{(2(-1.7)^2 - 4(-1.7) + 3)}{6(-1.7)^2 - 4} f(x) = 2x^2 - 4x + 3$
 $f'(x) = 6x^2 - 4$ (M1) (B1)

$x_1 = -1.6981$

$x_2 = -1.6981 - \frac{(2(-1.6981)^2 - 4(-1.6981) + 3)}{6(-1.6981)^2 - 4}$ (M1)

$x_2 = -1.6980$

$|-1.6980 - (-1.6981)| = 0.0001 < 0.0005$ (M1)

$x_{n+1} = -1.698$ (A)

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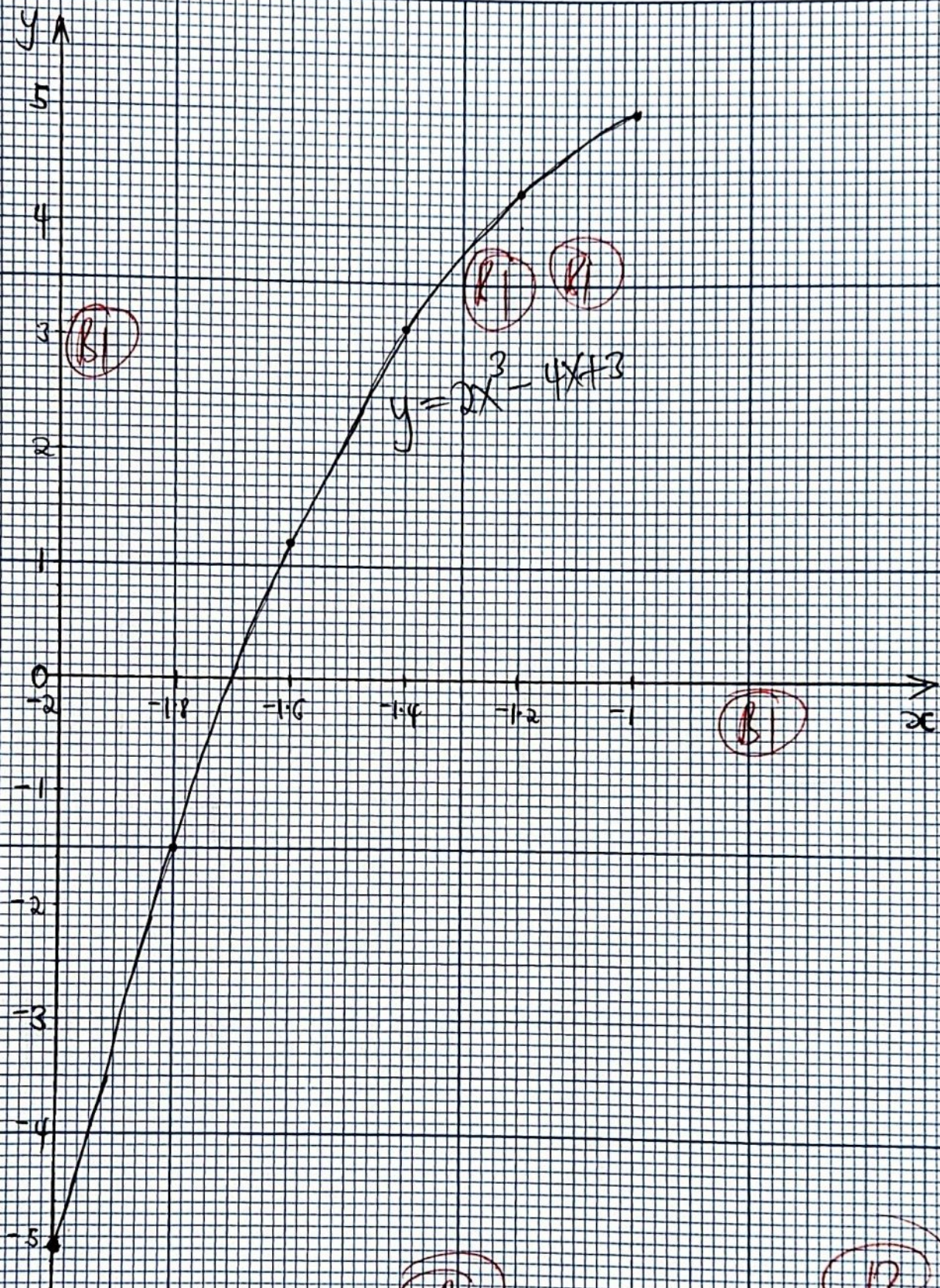


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$x_0 = -1.7$

B1

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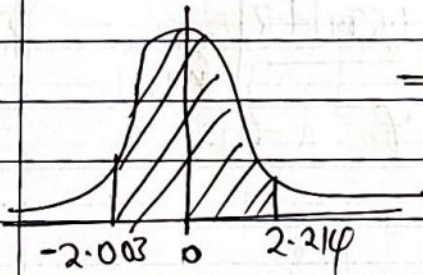
(15). $n = 120$, $p = \frac{1}{4}$, $q = \frac{3}{4}$

$\mu = 120 \times \frac{1}{4} = 30$ (B1), $\sigma = \sqrt{120 \times \frac{3}{4}} = \frac{3\sqrt{10}}{2}$ (B1)

a) (i) $P(20 < X \leq 40)$

$= P\left(\frac{20+0.5-30}{\frac{3\sqrt{10}}{2}} < Z < \frac{40+0.5-30}{\frac{3\sqrt{10}}{2}}\right)$

$= P(-2.003 < Z < 2.214)$

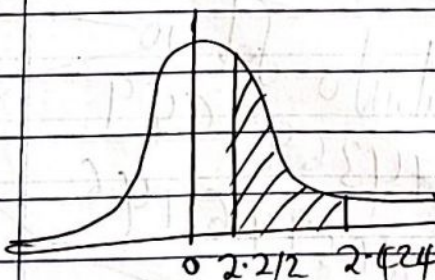


$= 0.4773 + 0.4865$ (M1)

$= 0.9638$ (A1)

(ii) $P(X = 41) = P\left(\frac{41-0.5-30}{\frac{3\sqrt{10}}{2}} < Z < \frac{41+0.5-30}{\frac{3\sqrt{10}}{2}}\right)$

$= P(2.214 < Z < 2.424)$



$= 0.4923 - 0.4865$ (M1)

$= 0.0058$ (A1)



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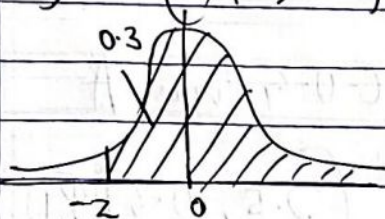
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b) $P(X \geq a) = 0.8$



$z = -0.842$ (B1)

$a - 0.5 - 30 = -0.842$ (mi)

$a = 26.51$

$a \approx 26$ (A7)

12



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16. a) let p be the mass/weight per unit area

Body	Area	Weight	C.O.G from A
ABCH	$5 \times 1.2 = 6$	$6p$	$(2.5, 0.6)$ (B1)
GDEF	$1 \times 3 = 3$	$3p$	$(2.5, 2.7)$ (B1)
ABCDEFGH	$6 + 3 = 9$	$9p$	$(\bar{x}, \bar{y})$ (B1)

$$9p \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = 6p \begin{pmatrix} 2.5 \\ 0.6 \end{pmatrix} + 3p \begin{pmatrix} 2.5 \\ 2.7 \end{pmatrix} \quad m_1 \quad m_2 \quad m_3$$

$$9 \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = \begin{pmatrix} 15 + 7.5 \\ 3.6 + 8.1 \end{pmatrix} = \begin{pmatrix} 22.5 \\ 11.7 \end{pmatrix} \quad m_1$$

$$\begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = \begin{pmatrix} 2.5 \\ 1.3 \end{pmatrix} m \quad (A1) \quad (A1)$$



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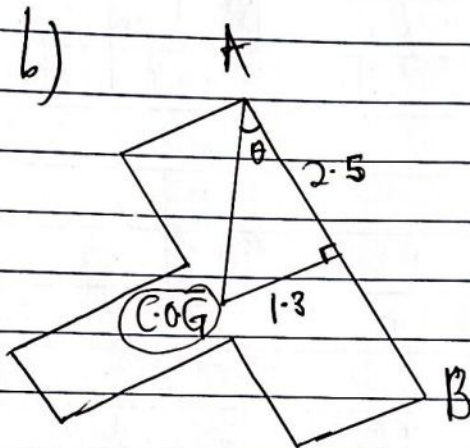
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$$\tan \theta = \frac{1.3}{2.5} \quad (B1)$$

$$\theta = \tan^{-1} \left(\frac{1.3}{2.5} \right) \quad (M1)$$

$$\theta = 27.47^\circ \quad (A1)$$

(12)

Mubir Johnson

0743649649

0703946946

St. Mary's College Kisubi

P425/1
PURE MATHEMATICS
Paper 1
Nov./Dec. 2023
3 hours



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Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

Answer all the eight questions in section A and any five from section B.

Any additional question(s) answered will not be marked.

All necessary working must be shown clearly.

Begin each answer on a fresh sheet of paper.

Graph paper is provided.

Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

SECTION A (40 MARKS)

Answer **all** the questions in this section.

1. Prove by induction that $\sum_{r=1}^n r^2 = \frac{n(n+1)(2n+1)}{6}$. (05 marks)
2. If a line $y = mx + c$ is a tangent to the curve $4x^2 + 3y^2 = 12$, show that $c^2 = 4 + 3m^2$. (05 marks)
3. Given that $y = e^x \cos 3x$, show that $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 10y = 0$. (05 marks)
4. Find the angle between the line $\mathbf{r} = \begin{pmatrix} 2 \\ 0 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 12 \\ 4 \end{pmatrix}$ and the plane $-x + 2y + 2z - 66 = 0$. (05 marks)
5. Solve the inequality $\frac{7-2x}{(x+1)(x-2)} > 0$. (05 marks)
6. Evaluate $\int_0^{\pi/3} (1 + \cos 3y)^2 dy$. (05 marks)
7. Express $2\sin\theta + 3\cos\theta$ in the form $R \sin(\theta + \alpha)$. (05 marks)
8. Use Maclaurin's theorem to expand $\ln(2+x)$, in ascending powers of x as far as the term in x^2 . (05 marks)

SECTION B (60 MARKS)

Answer any five questions from this section. All questions carry equal marks.

9. (a) Solve the equation $Z^3 - 7Z^2 + 19Z - 13 = 0$. (06 marks)
(b) Find the fourth roots of $8(-\sqrt{3} + i)$. (06 marks)
10. Express $f(x) = \frac{3x^3 + 2x^2 - 3x + 1}{x(1-x)}$ in partial fractions.
Hence find $\int f(x) dx$. (12 marks)
11. A point E has coordinates $(2, 0, -1)$. A line through E and parallel to the line whose equation is $\frac{x}{-2} = y = \frac{z+1}{2}$, meets a plane $x + 2y - 2z = 8$ at a point B . A perpendicular line from E meets the plane at a point C . Determine the coordinates of;
(a) B . (07 marks)
(b) C . (05 marks)
12. (a) Four different Mathematics books and six other different books are to be arranged on a shelf. In how many ways can the Mathematics books be arranged on the shelf? (02 marks)
(b) On a certain day, Fatuma drunk 6 bottles of the 9 bottles of soda available. On the next day she drunk 5 bottles of the 7 bottles of soda available. In how many ways could she have chosen the bottles of soda to drink in the two days? (03 marks)
(c) Given that ${}^{20}C_r = {}^{20}C_{r-2}$, find the value of r . (07 marks)
13. (a) A curve is given by the parametric equations $x = t^2 - 3$, $y = t(t^2 - 3)$. Find the Cartesian equation of the curve. (04 marks)
(b) A point P is such that its distance from the origin is five times its distance from $(12, 0)$.
(i) Show that the locus of P is a circle.
(ii) Determine the coordinates of the centre of the circle and its radius. (08 marks)

14. Given the curve $y = \frac{1}{4x^2 - 1}$, determine the;

(a) coordinates of the turning points of the curve. (03 marks)

(b) equation of the asymptotes.
Hence sketch the curve. (09 marks)

15. (a) Show that $\tan 3\theta = \frac{\tan \theta (3 - \tan^2 \theta)}{(1 - 3 \tan^2 \theta)}$. (05 marks)

(b) Solve the equation $\cos 4x + \cos 6x + \cos 2x = 0$ for $0^\circ \leq x \leq 180^\circ$. (07 marks)

16. The rate at which a body cools is proportional to the amount by which its temperature exceeds that of its surroundings. The body is placed in a room of temperature 25°C . After 6 minutes the temperature of the body dropped from 90°C to 60°C .

(a) Form a differential equation for the rate of cooling of the body. (07 marks)

(b) Find the time it takes for the body to cool from 40°C to 30°C . (05 marks)

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①. $1^2 + 2^2 + \dots + n^2 = \frac{n}{6}(n+1)(2n+1)$

LHS

$n=1 \quad 1^2 = 1$

RHS

$\frac{1}{6}(1+1)(2+1) = \frac{6}{6} = 1 \quad B_1$

$n=2 \quad 1^2 + 2^2 = 5$

$\frac{2}{6}(2+1)(4+1) = 5$

$n=k$

$\sum_{r=1}^k r^2 = \frac{k}{6}(k+1)(2k+1)$

B_1

$n=k+1$

LHS = $\sum_{r=1}^{k+1} r^2 = \frac{k}{6}(k+1)(2k+1) + (k+1)^2$

M_1

$= \frac{(k+1)}{6}(2k^2 + k + 6k + 6)$

$= \frac{(k+1)}{6}(2k^2 + 7k + 6)$

$= \frac{(k+1)}{6}(2k^2 + 4k + 3k + 6)$

M_1

$= \frac{(k+1)}{6}[2k(k+2) + 3(k+2)]$

$= \frac{(k+1)}{6}(k+2)(2k+3)$

B_1

RHS for $n=k+1$

$= \frac{(k+1)}{6}(k+1+1)[2(k+1)+1] = \frac{(k+1)}{6}(k+2)(2k+3)$



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②. $y = mx + c$ $4x^2 + 3y^2 = 12$
 $4x^2 + 3(mx + c)^2 = 12$

$$4x^2 + 3m^2x^2 + 6mcx + 3c^2 - 12 = 0 \quad M1$$

$$(4 + 3m^2)x^2 + 6mcx + 3c^2 - 12 = 0$$

$$(6mc)^2 = 4(4 + 3m^2)(3c^2 - 12) = 0 \quad M1$$

$$36m^2c^2 = 4(12c^2 - 48 + 9m^2c^2 - 36m^2) \quad M1$$

$$9m^2c^2 = 12c^2 - 48 + 9m^2c^2 - 36m^2$$

$$12c^2 = 48 + 36m^2 \quad M1$$

$$\underline{c^2 = 4 + 3m^2} \quad B1$$

③. $y = e^x \cos 3x$

$$\frac{dy}{dx} = e^x \cos 3x - 3e^x \sin 3x = y - 3e^x \sin 3x \quad M1 \ A1$$

$$\frac{d^2y}{dx^2} = \frac{dy}{dx} - 3e^x \sin 3x - 9e^x \cos 3x \quad M1$$

$$\frac{d^2y}{dx^2} = \frac{dy}{dx} + \frac{dy}{dx} - y - 9y \quad M1$$

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 10y = 0 \quad B1$$



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④. $\begin{pmatrix} 3 \\ 12 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix} = \sqrt{3^2+12^2+4^2} \sqrt{(-1)^2+2^2+2^2} \sin \alpha$ B1 M1

$-3+24+8 = \sqrt{169} \sqrt{9} \sin \alpha$ M1

$29 = 13 \times 3 \sin \alpha$

$\alpha = \sin^{-1} \left(\frac{29}{39} \right)$ $\alpha = 48.04^\circ$ M1 A1

⑤. Critical values

$x = -1, 2, \frac{7}{2}$ B1

	$x < -1$	$-1 < x < 2$	$2 < x < \frac{7}{2}$	$x > \frac{7}{2}$
$\frac{7-2x}{(x+1)(x-2)}$	+	-	+	-

M1 M1

$x < -1$ and $2 < x < \frac{7}{2}$ A1 A1

⑥. $\int_0^{\pi/3} (1 + \cos 3y)^2 dy = \int_0^{\pi/3} (1 + 2\cos 3y + \cos^2 3y) dy$

$= \int_0^{\pi/3} \left(\frac{3}{2} + 2\cos 3y + \frac{1}{2} \cos 6y \right) dy$ M1

$= \frac{3}{2} y \Big|_0^{\pi/3} + \frac{2}{3} \sin 3y \Big|_0^{\pi/3} + \frac{1}{12} \sin 6y \Big|_0^{\pi/3}$ M1 A1

$= \frac{3}{2} \left(\frac{\pi}{3} - 0 \right) + \frac{2}{3} (\sin \pi - \sin 0) + \frac{1}{12} (\sin 2\pi - \sin 0)$ M1

$= \frac{\pi}{2}$ or $\frac{22}{7}$ A1

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7. $2\sin\theta + 3\cos\theta \equiv R\sin(\theta + \alpha) \equiv R\sin\theta\cos\alpha + R\cos\theta\sin\alpha$ M1

$R = \sqrt{2^2 + 3^2} = \sqrt{13}$ A1

$\alpha = \cos^{-1}\left(\frac{2}{\sqrt{13}}\right) = \sin^{-1}\left(\frac{3}{\sqrt{13}}\right) = \tan^{-1}\left(\frac{3}{2}\right) = 56.31^\circ$ M1 A1

$2\sin\theta + 3\cos\theta = \sqrt{13} \sin(\theta + 56.31^\circ)$ B1

8. $f(x) = \ln(2+x)$, $f(0) = \ln 2$ B1

$f'(x) = \frac{1}{2+x}$, $f'(0) = \frac{1}{2}$ B1

$f''(x) = \frac{-1}{(2+x)^2}$, $f''(0) = -\frac{1}{4}$ B1

$\ln(2+x) = \ln 2 + \frac{1}{2}x - \frac{1}{4}x^2$ M1

$= \ln 2 + \frac{1}{2}x - \frac{1}{8}x^2$ A1



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9. a) $z^3 - 7z^2 + 19z - 13 = 0$

	z^3	z^2	z	z^0
	1	-7	19	-13
$z=1$	0	1	-6	13
	1	-6	13	0

M1

$$z^2 - 6z + 13 = 0$$

$$(z-3)^2 - 9 + 13 = 0 \quad (z-3)^2 = -4 \quad M1$$

$$z-3 = \sqrt{-4} = \pm 2i \quad M1 \quad z = 1, 3 \pm 2i \quad A1 A1 A1$$

b) $z = 8(-\sqrt{3} + i) \quad |z| = \sqrt{8^2 + (-8\sqrt{3})^2} = 16$

$$\text{Arg}(z) = 180^\circ - \tan^{-1}\left(\frac{8}{8\sqrt{3}}\right) = 150^\circ \quad B1$$

$$z = 16 (\cos 150^\circ + i \sin 150^\circ)$$

$$z = 16 \left\{ \cos \left(\frac{150^\circ + 360^\circ k}{4} \right) + i \sin \left(\frac{150^\circ + 360^\circ k}{4} \right) \right\} \quad M1$$

$$k=0 \quad z_1 = 2 (\cos 37.5^\circ + i \sin 37.5^\circ) = 1.5867 + 1.2175i \quad A1$$

$$k=1 \quad z_2 = 2 (\cos 127.5^\circ + i \sin 127.5^\circ) = -1.2175 + 1.5867i \quad A1$$

$$k=2 \quad z_3 = 2 (\cos 217.5^\circ + i \sin 217.5^\circ) = -1.5867 - 1.2175i \quad A1$$

$$k=3 \quad z_4 = 2 (\cos 307.5^\circ + i \sin 307.5^\circ) = 1.2175 - 1.5867i \quad A1$$



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10. $f(x) = \frac{3x^3 + 2x^2 - 3x + 1}{x(1-x)} \equiv \frac{Ax+B}{x} + \frac{C}{1-x} + \frac{D}{1-x}$ M1

$$3x^3 + 2x^2 - 3x + 1 \equiv (Ax+B)x(1-x) + C(1-x) + Dx$$
 M1

$x=0 \quad 1=C$ A1 $x=1 \quad 3+2-3+1=D \quad D=3$ A1

$(x^3) \quad 3 = -A \quad A = -3$ A1

$(x^2) \quad A-B=2 \quad B = -3-2 = -5$ A1

$$\frac{3x^3 + 2x^2 - 3x + 1}{x(1-x)} = \frac{-3x-5}{x} + \frac{1}{x} + \frac{3}{1-x}$$
 A1

$$\int f(x) dx = \int \left\{ \frac{-3x-5}{x} + \frac{1}{x} + \frac{3}{1-x} \right\} dx$$

$$= \frac{-3x^2}{2} - 5x + \ln x - 3 \ln(1-x) + k$$

M1 M1 M1 M1 A1



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①. $E(2, 0, -1)$ $\underline{d} = \begin{pmatrix} -2 \\ 1 \\ 2 \end{pmatrix}$ B1

a) line $\underline{r} = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} + \alpha \begin{pmatrix} -2 \\ 1 \\ 2 \end{pmatrix}$ M1 A1
 plane $X + 2y - 2z = 8$

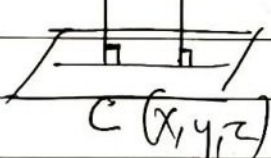
$$2 - 2\alpha + 2\alpha - 2(-1 + 2\alpha) = 8 \quad M1$$

$$2 + 2 - 4\alpha = 8$$

$$-4\alpha = 4$$

$$\alpha = -1 \quad B1$$

At B $\underline{r} = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} - \begin{pmatrix} -2 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 4 \\ -1 \\ -3 \end{pmatrix}$ M1 B $(4, -1, -3)$ A1

b) $E(2, 0, -1)$
 $\underline{n} = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$ $\overrightarrow{EC} = k\underline{n}$

 $\begin{pmatrix} x-2 \\ y \\ z+1 \end{pmatrix} = \begin{pmatrix} k \\ 2k \\ -2k \end{pmatrix}$ M1

$$x = 2 + k, y = 2k, z = -1 - 2k$$

$$2 + k + 2(2k) - 2(-1 - 2k) = 8 \quad M1$$

$$2 + k + 4k + 2 + 4k = 8$$

$$9k = 4 \quad k = \frac{4}{9} \quad B1$$

$$x = 2 + \frac{4}{9} = \frac{22}{9}, y = 2\left(\frac{4}{9}\right) = \frac{8}{9}, z = -1 - 2\left(\frac{4}{9}\right) = -\frac{17}{9} \quad M1$$

$C\left(\frac{22}{9}, \frac{8}{9}, -\frac{17}{9}\right)$



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② a) $4 \times 3 \times 2 \times 1 = 4! = 24$

m | A

b) ${}^9C_6 \times {}^7C_5 = 84 \times 21 = 1764$

m | m | A

c) ${}^{20}C_r = {}^{20}C_{r-2}$

$\frac{20!}{(20-r)!r!} = \frac{20!}{(20-r+2)!(r-2)!}$

m | m |

$\frac{(20-r)!}{(r)(r-1)} \cdot \frac{(r-2)!}{(r-2)!} = \frac{(20-r)!}{(r-2)!}$

m |

$r(r-1)(20-r)! = (20-r)(20-r-1)(20-r)! \quad m |$

$r^2 - r = 462 - 22r - 21r + r^2$

m | m |

$43r - r = 462$

$42r = 462$

$r = 11$

A



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⑬ a) $x = t^2 - 3, y = t(t^2 - 3) \Rightarrow y = tx$

$t = \frac{y}{x}$ B1 $x = \left(\frac{y}{x}\right)^2 - 3$ M1

$y^2 = x^3 + 3x^2$ M1 A1

b) P(x, y) O(0, 0) A(12, 0)

(i) $PO = 5PA \quad \sqrt{(x-0)^2 + (y-0)^2} = 5\sqrt{(x-12)^2 + (y-0)^2}$ M1

$x^2 + y^2 = 25(x^2 - 24x + 144 + y^2)$ M1 M1

$24x^2 + 24y^2 - 600x + 3600 = 0$

$x^2 + y^2 - 25x + 150 = 0$ B1

(ii) $\left(x - \frac{25}{2}\right)^2 + y^2 - \frac{625}{4} + 150 = 0$ M1

$\left(x - \frac{25}{2}\right)^2 + y^2 = \frac{25}{4}$ A1

Centre $\left(\frac{25}{2}, 0\right)$ A1 $r^2 = \frac{25}{4}$ radius = $\frac{5}{2}$ A1



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14. a) $y = \frac{1}{4x^2-1}$ $\frac{dy}{dx} = \frac{-8x}{(4x^2-1)^2} = 0$ $x=0$ $y(0) = -1$ $(0, -1)$ A_1

$\frac{d^2y}{dx^2} = \frac{-8(4x^2-1)^2 + 128x^2(4x^2-1)}{(4x^2-1)^4}$ $\frac{d^2y}{dx^2} \bigg|_{x=0} = -8 < 0 \Rightarrow \pi$

~~At~~ $4yx^2 - (y+1) = 0$ $0 + 4(4y)(y+1) < 0$
 $y(y+1) < 0$ $y = 0, -1$

	$y < -1$	$-1 < y < 0$	$y > 0$
$y(y+1)$	+	-	+

Curve doesn't lie $-1 < y < 0$

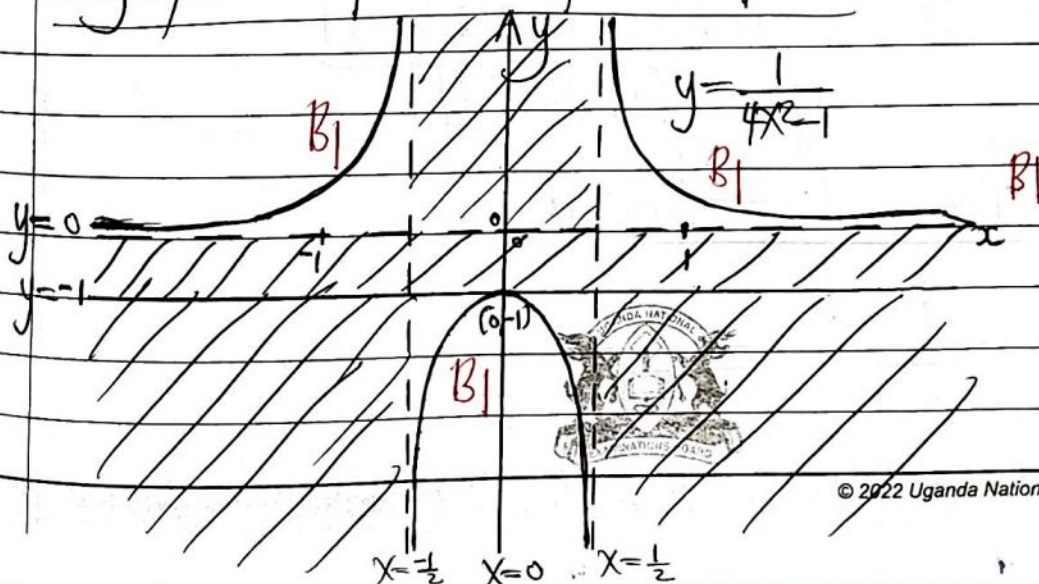
b) Asymptotes as $x \rightarrow \infty$ $y = 0$ B_1 as $y \rightarrow \infty$ $y = \frac{1}{(2x+1)(2x-1)}$

$2x \pm 1 = 0$

$x = \pm \frac{1}{2}$ $B_1 B_1$

Intercepts $(0, -1)$ B_1

	$x < -\frac{1}{2}$	$-\frac{1}{2} < x < 0$	$0 < x < \frac{1}{2}$	$x > \frac{1}{2}$
y	+	-	-	+



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(15) a) $\tan 3\theta = \tan(2\theta + \theta) = \frac{\tan 2\theta + \tan \theta}{1 - \tan \theta \tan 2\theta}$ M1

$$= \frac{2\tan \theta + \tan \theta}{1 - \tan^2 \theta}$$

$$= \frac{2\tan \theta + \tan \theta - \tan^3 \theta}{1 - \tan^2 \theta - 2\tan^2 \theta}$$

$$\tan 3\theta = \frac{3\tan \theta - \tan^3 \theta}{1 - 3\tan^2 \theta} = \frac{\tan \theta (3 - \tan^2 \theta)}{1 - 3\tan^2 \theta}$$

Alt

$$\tan 3\theta = \frac{\sin 3\theta}{\cos 3\theta} = \frac{3\sin \theta - 4\sin^3 \theta}{4\cos^3 \theta - 3\cos \theta}$$

$$= \frac{3\tan \theta \sec^2 \theta - 4\tan^3 \theta}{4 - 3\sec^2 \theta}$$

$$= \frac{3\tan \theta (1 + \tan^2 \theta) - 4\tan^3 \theta}{4 - 3(1 + \tan^2 \theta)}$$

$$\tan 3\theta = \frac{\tan \theta (3 - \tan^2 \theta)}{1 - 3\tan^2 \theta}$$



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$$b) \cos 6X + \cos 2X + \cos 4X = 0$$

$$2 \cos 4X \cos 2X + \cos 4X = 0 \quad m1$$

$$(\cos 4X)(2 \cos 2X + 1) = 0 \quad m1$$

$$\cos 4X = 0$$

$$\cos 2X = -\frac{1}{2} \quad A1$$

$$4X = 90^\circ, 270^\circ, 450^\circ, 630^\circ \quad m1 \quad 2X = 120^\circ, 240^\circ \quad m1$$

$$X = 22.5^\circ, 60^\circ, 67.5^\circ, 112.5^\circ, 120^\circ, 157.5^\circ \quad A1 \quad A1$$

Alt

$$4 \cos^3 2X - 3 \cos 2X + 2 \cos^2 2X - 1 + \cos 2X = 0$$

$$4 \cos^3 2X + 2 \cos^2 2X - 2 \cos 2X - 1 = 0$$

$$2 \cos^2 2X (2 \cos 2X + 1) - (2 \cos 2X + 1) = 0$$

$$(2 \cos 2X + 1)(2 \cos^2 2X - 1) = 0$$

$$\cos 2X = -\frac{1}{2}, \pm \frac{1}{\sqrt{2}}$$

$$2X = 45^\circ, 120^\circ, 135^\circ, 225^\circ, 240^\circ, 315^\circ$$

$$X = 22.5^\circ, 60^\circ, 67.5^\circ, 112.5^\circ, 120^\circ, 157.5^\circ$$



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(16) a) $-\frac{d\theta}{dt} \propto (\theta - 25)$ M1 $\frac{d\theta}{dt} = -k(\theta - 25)$ B1

$$\int \frac{d\theta}{\theta - 25} = \int -k dt$$
 M1

$$\ln(\theta - 25) + c_1 = -kt + c_2$$

$$\ln(\theta - 25) = -kt + c, \quad c = c_2 - c_1$$

At $t = 0, \theta = 90$, $t = 6, \theta = 60$

$$\ln(90 - 25) = c \quad c = \ln 65$$
 B1

$$\ln(60 - 25) = -6k + \ln 65$$

$$6k = \ln 65 - \ln 35 = \ln\left(\frac{65}{35}\right)$$

$$k = \frac{1}{6} \ln\left(\frac{13}{7}\right)$$
 B1

$$\frac{t}{6} \ln\left(\frac{13}{7}\right) = \ln 65 - \ln(\theta - 25)$$
 M1

$$\frac{t}{6} \ln\left(\frac{13}{7}\right) = \ln\left(\frac{65}{\theta - 25}\right)$$
 A1



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b) $t_1 = 40$, $t_1 = ?$
 $t_2 = 30$, $t_2 = ?$

Mubiko John
0703 94 6946

$$\frac{t_1}{6} \ln \left(\frac{13}{7} \right) = \ln \left(\frac{65}{40-25} \right) = \ln \left(\frac{13}{3} \right)$$

$$t_1 = \frac{6 \ln \left(\frac{13}{3} \right)}{\ln \left(\frac{13}{7} \right)} = 14.21238 \quad M1 \quad B1$$

$$\frac{t_2}{6} \ln \left(\frac{13}{7} \right) = \ln \left(\frac{65}{30-25} \right) = \ln 13$$

$$t_2 = \frac{6 \ln 13}{\ln \left(\frac{13}{7} \right)} = 24.86061 \quad B1$$

$$t = t_2 - t_1 = 24.86061 - 14.21238 \quad M1$$

$$t = 10.64823 \approx 10.65 \text{ min}$$

$$0.65 \times 60 = 39s$$

$$t = \underline{10 \text{ min } 39s} \quad A1$$



P425/2
APPLIED MATHEMATICS
Paper 2
Nov./Dec. 2023
3 hours



UGANDA NATIONAL EXAMINATIONS BOARD
Uganda Advanced Certificate of Education
APPLIED MATHEMATICS

Paper 2

3 hours

INSTRUCTIONS TO CANDIDATES:

*Answer **all** the **eight** questions in section A and any **five** from section B.*

*Any additional question(s) answered will **not** be marked.*

***All** necessary working **must** be shown clearly.*

Begin each answer on a fresh sheet of paper.

Graph paper is provided.

Silent, non programmable scientific calculators and mathematical tables with a list of formulae may be used.

In numerical work, take acceleration due to gravity g , to be 9.8 ms^{-2} .

SECTION A (40 MARKS)

Answer all the questions in this section.

1. A coin is biased such that when it is tossed the head is twice as likely to occur as the tail. Find the probability that in seven tosses, there will be exactly two tails. (05 marks)
2. Two bodies A and B of masses 6 kg and 2 kg moving along a straight line with velocities 4 ms^{-1} and 2 ms^{-1} respectively, collide head on. After collision, A moves with a velocity of 2.6 ms^{-1} in the same direction.
Calculate the;
(a) velocity of B after collision. (02 marks)
(b) loss in kinetic energy. (03 marks)
3. The values of a function $f(x)$ are given in the table below.

x	0	$\frac{1}{2}$	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3
$f(x)$	0.1003	0.0391	0.0801	0.0602	0.0649	0.0380	0.0327

Use the trapezium rule to estimate the value of

$$\int_0^3 f(x) dx$$

correct to **three** decimal places.

(05 marks)

4. A ball of mass 1 kg rolls from rest down a rough plane inclined at 30° to a horizontal ground. The ball rolls for 4 m before it reaches the ground. The coefficient of friction between the ball and the plane is $\frac{1}{4}$. Find the velocity with which the ball reaches the ground. (05 marks)
5. The table below shows the age distribution of a population of a certain town in a census.

AGE (years)	NUMBER ('000)
Under 10	15
10 and under 20	19
20 and under 30	16
30 and under 40	18
40 and under 60	30
60 and under 80	6
80 and under 90	1

- (a) Draw a histogram for the data.

(03 marks)

(b) Use the histogram to estimate the modal age of the population.

(02 marks)

6. The numbers $x = 6.45$, $y = 0.00215$ and $z = 2.7$ are each rounded off to the given number of decimal places.

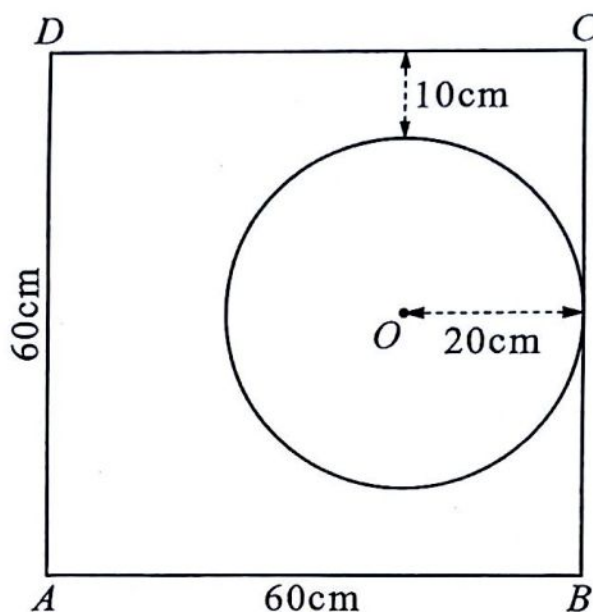
Determine the interval in which $w = \frac{x+z^3}{\sqrt{y}}$ lies. (05 marks)

7. Two independent events R and S are such that $P(R) = \frac{3}{4}$ and $P(S) = P(S' \cap R')$. Find;

(a) $P(S)$. (03 marks)

(b) $P(S' \cap R)$. (02 marks)

8. A uniform lamina in form of a square with side 60 cm has a circular hole of radius 20 cm made in it as shown in the diagram below.



Find the position of the centre of gravity of the lamina from side AD .

(05 marks)

SECTION B (60 MARKS)

Answer any five questions from this section. All questions carry equal marks.

9. The table below shows the scores of 10 candidates in Biology and Economics.

CANDIDATE	A	B	C	D	E	F	G	H	I	J
BIOLOGY	45	63	56	61	75	83	73	50	77	70
ECONOMICS	90	64	76	70	55	53	62	85	53	62

- (a) (i) Plot a scatter diagram for the data.
 (ii) Draw a line of best fit on the scatter diagram.
 (iii) Use your line of best fit to estimate the Biology mark for a candidate who scored 57 in Economics. (06 marks)
- (b) Calculate a rank correlation coefficient between the candidates' performance in the two subjects.
 Comment on your result. (06 marks)
10. Two points A and B are 526 m apart along a straight road. A car moving along the road passes point A with a constant speed of 25 ms^{-1} . The car maintains this speed for 10 seconds and then decelerates uniformly for 8 seconds until it attains a speed of $V \text{ ms}^{-1}$. The car maintains this speed until it passes point B . The total time taken by the car to move from point A to B is 30 seconds.
- (a) Sketch a Velocity – Time graph for the motion of the car. (04 marks)
- (b) Determine the;
 (i) value of V . (05 marks)
 (ii) deceleration of the car. (03 marks)
11. Given that $f(x) = xe^x + 5x - 10$;
- (a) (i) Evaluate $f(1)$ and $f(2)$, correct to **four** decimal places.
 (ii) Deduce that the equation $f(x) = 0$ has a root between $x = 1$ and $x = 2$. (04 marks)
- (b) Use linear interpolation **twice** to obtain the root of the equation $xe^x + 5x - 10 = 0$, correct to **three** decimal places. (08 marks)

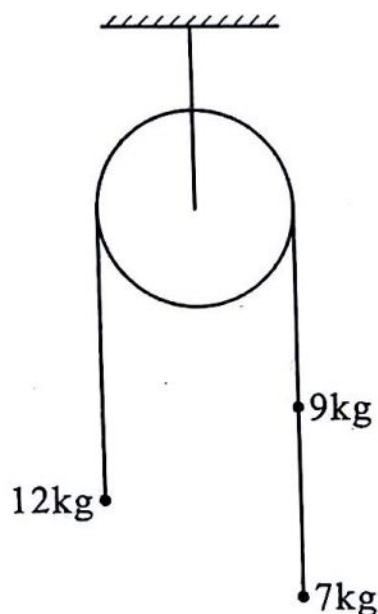
12. A continuous random variable X has a cumulative distribution function given by

$$F(x) = \begin{cases} \frac{1+x}{6}, & -1 \leq x \leq 0 \\ \frac{1+2x}{6}, & 0 \leq x \leq 2 \\ \frac{4+3x}{12}, & 2 \leq x \leq \frac{8}{3} \\ 1, & x \geq \frac{8}{3} \end{cases}$$

Find;

- (a) the median. (03 marks)
 - (b) the probability density function $f(x)$. (03 marks)
 - (c) $P(1 \leq X \leq 2.5)$. (03 marks)
 - (d) the mean of X . (03 marks)
13. Three forces $F_1 = (2\mathbf{i} - 3\mathbf{j})$ N, $F_2 = (5\mathbf{i} + 2\mathbf{j})$ N and $F_3 = (-2\mathbf{i} - 11\mathbf{j})$ N act at points (2, 3), (-2, 3) and (3, -2) respectively. Determine the;
- (a) magnitude of their resultant force. (03 marks)
 - (b) equation of the line of action of the resultant force. (05 marks)
 - (c) distance from the origin at which the resultant cuts the x -axis. (02 marks)
 - (d) force that should be added to form a couple. (02 marks)
14. (a) Show that the formula based on Newton Raphson method for approximating the k^{th} root of a number N is given by
- $$x_{n+1} = \frac{(k-1)x_n^k + N}{kx_n^{k-1}} \quad n = 0, 1, 2, 3, \dots$$
- (04 marks)
- (b) Construct a flow chart that;
 - (i) reads in the initial approximation x_0 , k and N ,
 - (ii) computes and prints N and its k^{th} root correct to **three** decimal places. (05 marks)
 - (c) Perform a dry run for your flow chart when $N = 13$, $x_0 = 1.6$ and $k = 4$. (03 marks)

15. (a) A woman travelling to work by a car goes through three police check points A , B and C . The probabilities that she is delayed at A , at B and at C are 0.3, 0.5 and 0.7 respectively.
Determine the probability that she is delayed at;
- (i) only one check point. (03 marks)
- (ii) two or more check points. (03 marks)
- (b) A man goes to work by route P or route Q . The probability that he takes route P is 0.6. The probability that he is late given that he goes through P is $\frac{2}{3}$ and through Q is $\frac{1}{3}$.
- (i) Find the probability that he is late for work on a certain day. (03 marks)
- (ii) Given that he is **not** late, determine the probability that he went through P . (03 marks)
16. The diagram below shows three masses of 12 kg, 9 kg and 7 kg connected by light inelastic strings. The string connecting the 12 kg and 9 kg masses passes over a smooth fixed pulley. The other string connects the 9 kg and 7 kg masses.



The system is released from rest and the 12 kg mass accelerates upwards.

- (a) Calculate the;
- (i) acceleration of the system.
- (ii) tensions in the strings. (10 marks)
- (b) Determine the velocity of the 12 kg mass after 1.5 seconds. (02 marks)

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①. $P(T) = p, P(H) = q = 2p, p + q = 1$

$p + 2p = 1$ M1 $p = \frac{1}{3}, q = \frac{2}{3}, n = 7$ B1

$P(X=2) = {}^7C_2 \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^5$
M1 M1 $= 21 \times \frac{1}{9} \times \frac{32}{243} = \frac{224}{729}$ A1
 OR 0.30727 [5]

②. a) $(6 \times 4) - (2 \times 2) = (6 \times 2.6) + 2V$ M1
 $20 = 15.6 + 2V$
 $V = 2.2 \text{ ms}^{-1}$ or $\frac{11 \text{ ms}^{-1}}{5}$ A1

b) Initial k.e = $\left(\frac{1}{2} \times 6 \times 4^2\right) + \left(\frac{1}{2} \times 2 \times 2^2\right) = 52 \text{ J}$ B1
 Final k.e = $\left(\frac{1}{2} \times 6 \times 2.6^2\right) + \left(\frac{1}{2} \times 2 \times 2.2^2\right) = 25.12 \text{ J}$
 loss in k.e = $52 - 25.12 = 26.88 \text{ J}$ M1 A1
[5]

③. $h = \frac{3-0}{7-1} = \frac{3}{6} = \frac{1}{2}$ B1
 $0.1003 + 0.0327 = 0.133$ B1
 $0.0391 + 0.0801 + 0.0602 + 0.0649 + 0.0380 = 0.2823$ B1
 $\int_0^3 f(x) dx = \frac{1}{2} \times \frac{1}{2} [0.133 + 2 \times 0.2823] = 0.1744$ M1
 ≈ 0.174 A1
[5]

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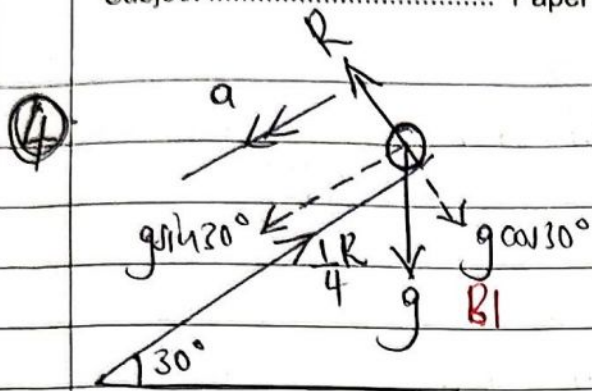
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$$g \sin 30^\circ - \frac{1}{4}R = a \quad M1$$

$$9.8 \sin 30^\circ - 0.25 \times 9.8 \cos 30^\circ = a$$

$$a = 2.7782 \text{ ms}^{-2} \quad A1$$

$$V^2 = u^2 + 2as \quad V = \sqrt{0^2 + 2 \times 2.7782 \times 4} \quad M1$$

$$V = 4.7144 \text{ ms}^{-1} \quad A1$$

Alt loss in p.e = w.d against friction + gain in k.e
 $mg d \sin \theta = \mu mg \cos \theta \times d + \frac{1}{2} m (V^2 - u^2)$

$$9.8 \times 4 \sin 30^\circ = \frac{1}{4} \times 9.8 \times 4 \cos 30^\circ + \frac{1}{2} V^2 \quad (M1) (M1) (M1)$$

$$V = 4.7144 \text{ ms}^{-1} \quad (A1) \quad [5]$$

⑤. c.b	0-10	10-20	20-30	30-40	40-60	60-80	80-90
f	15	19	16	18	30	6	1
l	10	10	10	10	20	20	10
f.d	1.5	1.9	1.6	1.8	1.5	0.3	0.1

a) see graph

B1 - correct values of f.d with/without $\times 1000$



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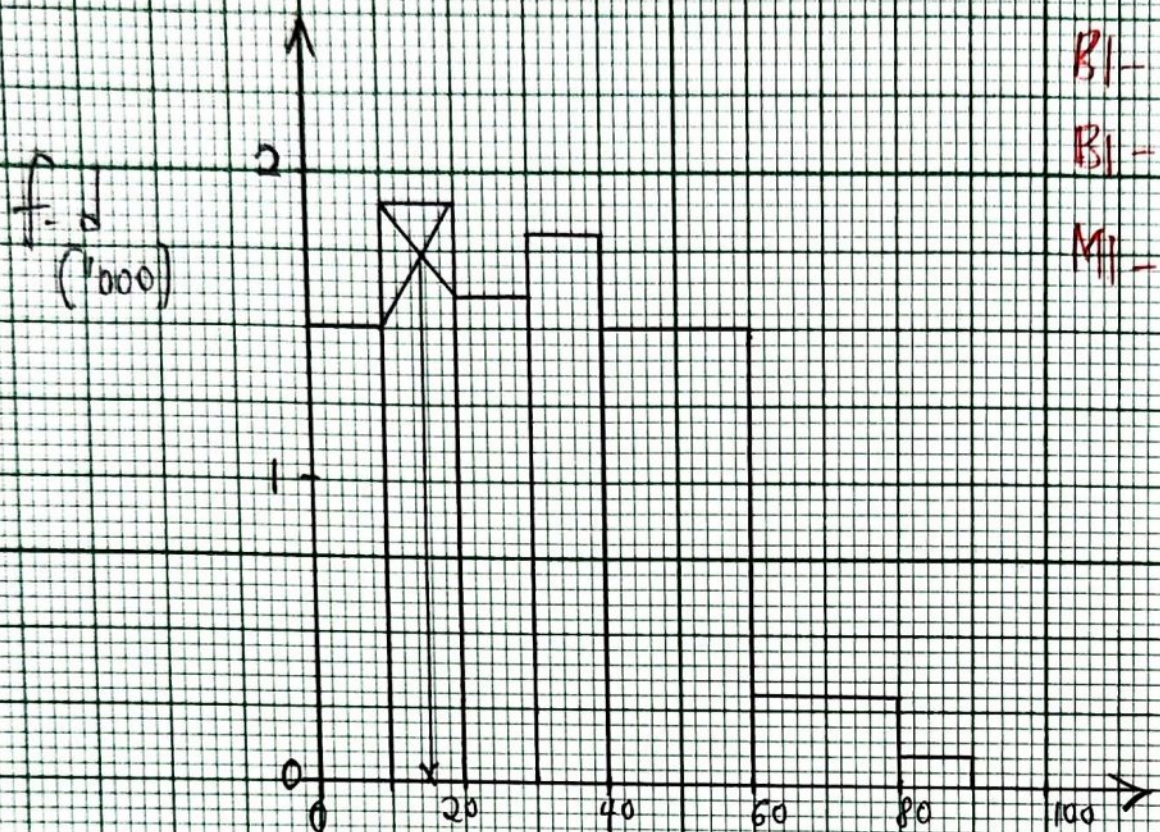
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BI - axes & scaling
 BI - plotting bars
 MI - labelling mode

Mode = 15 A1

Age (years)
 /c.b

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$$\textcircled{6}. \left(\frac{x+z^3}{\sqrt{y}} \right)_{\max} = \frac{6.455 + (2.75)^3}{\sqrt{0.002145}} \quad \text{M1} \quad \text{B1} = 588.4137$$

$$\left(\frac{x+z^3}{\sqrt{y}} \right)_{\min} = \frac{6.455 + (2.75)^3}{-\sqrt{0.002145}} \quad \text{M1} \quad \text{B1} = -588.4137$$

$$\underline{[-588.4137, 588.4137]}$$

A1

5

$$\textcircled{7}. a) P(R) = \frac{3}{4}, P(R') = \frac{1}{4}$$

$$P(S) = P(R') \times P(S') = \frac{1}{4} P(S') \quad \text{B1}$$

$$P(S) + P(S') = 1$$

$$P(S) + 4P(S) = 1 \quad \text{M1} \quad P(S) = \frac{1}{5} \quad \text{A1}$$

$$b) P(S' \cap R) = P(S') \times P(R)$$

$$= \frac{4}{5} \times \frac{3}{4} \quad \text{M1}$$

$$= \frac{3}{5}$$

A1

5



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⑧ p - weight per unit area

Body

Height

C.O.G from AD



60x60p

30

B1



$\frac{22}{7} \times 20 \times 20p$

$20 + 20 = 40$

B1



$3600p - \frac{8800p}{7} = \frac{16400p}{7}$

$\bar{x}$

B1

$$\uparrow \quad \frac{16400p}{7} \bar{x} = (3600p \times 30) - \left(\frac{8800p}{7} \times 40 \right) \quad m$$

$$\frac{164}{7} \bar{x} = \frac{1080}{7} - \frac{3520}{7} = \frac{4040}{7}$$

$$\bar{x} = \frac{4040}{164} = \frac{1010}{41} \text{ cm} = \underline{\underline{24.6341 \text{ cm}}} \quad A7$$

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⑨. a) (i) (ii) (iii) See graph

b)	r_1	10	6	8	7	3	1	4	9	2	5	MI
	r_2	1	5	3	4	8	9.5	6.5	2	9.5	6.5	MI
	d^2	81	1	25	9	25	72.25	6.25	49	56.25	2.25	

$$\sum d^2 = 327 \quad B1$$

Alt

r_1	1	5	3	4	8	10	7	2	9	6	
r_2	10	6	8	7	3	1.5	4.5	9	1.5	4.5	
d^2	81	1	25	9	25	72.5	6.25	49	56.25	2.25	

$$\sum d^2 = 327$$

$$r = 1 - \left(\frac{6 \times 327}{10^3 - 10} \right) = -0.9818 \quad MI \quad A1$$

$$\approx -0.982$$

negative very high coefficient B1

OR significant at 1% or 5%



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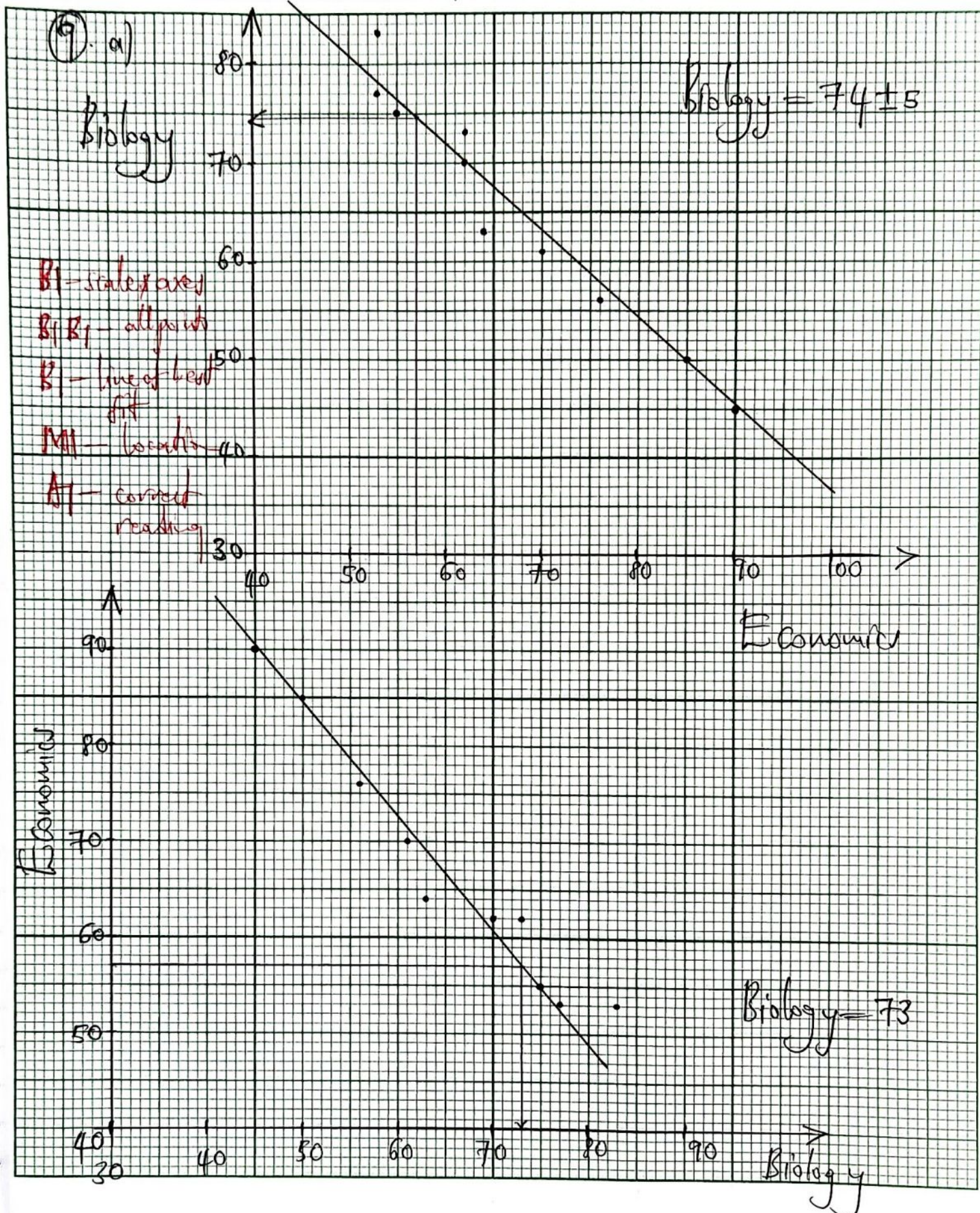
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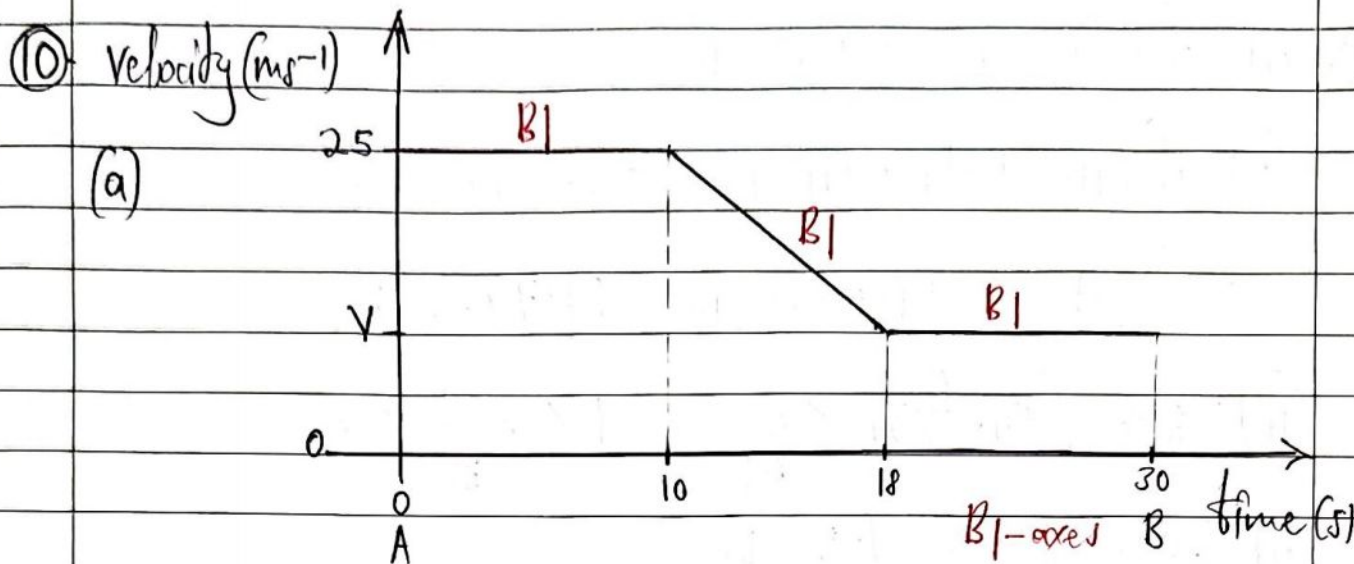
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(b) (i) $(25 \times 10) + \frac{1}{2} \times 8(25 + V) + 12V = 526$ m1 m1 m1 m1

$$250 + 100 + 4V + 12V = 526$$

$$16V = 176$$

$$V = 11 \text{ ms}^{-1}$$
 A1 with/without units

(ii) $\alpha = \frac{11 - 25}{8} = -\frac{7}{4} = -1.75 = -\frac{3}{4}$ m1 A1

Deceleration = 1.75 ms^{-2} B1

12



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11. a) $f(x) = xe^x + 5x - 10$

(i) $f(1) = 1e^1 + 5(1) - 10 = -2.2817$ M1

$f(2) = 2e^2 + 5(2) - 10 = 14.7781$ M1

(ii) since $f(1) \cdot f(2) < 0 \Rightarrow 1 < \text{root} < 2$
M1 B1

b) $x_0 = \frac{1+2}{2} = 1.5$ B1

$f(1.5) = 1.5e^{1.5} + 5(1.5) - 10 = 4.2225$ M1

1	x_1	1.5	
-2.2817	0	4.2225	B1

$\frac{1.5 - x_1}{4.2225 - 0} = \frac{1.5 - 1}{4.2225 - (-2.2817)}$ M1

$x_1 = 1.1754$ A1

$f(1.1754) = 1.1754e^{1.1754} + 5(1.1754) - 10 = -0.3154$ M1

1.1754	x_2	1.5	
-0.3154	0	4.2225	

$\frac{1.5 - x_2}{4.2225 - 0} = \frac{1.5 - 1.1754}{4.2225 - (-0.3154)}$ M1

$x_2 = 1.1980$

$x_{n+1} = 1.198$ A1



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12. a) $F(0) = \frac{1}{6} < \frac{1}{2}$ $F(2) = \frac{5}{6} > \frac{1}{2}$ $0 < m < 2$ M1 - finding

$F(m) = \frac{1}{2}$ $\frac{1+2m}{6} = \frac{1}{2}$ M1 $1+2m=3$
 $2m=2$

$m=1$ A1

b) $F(x) = \frac{1+x}{6}$, $F(x) = \frac{1+2x}{6}$, $F(x) = \frac{4+3x}{12}$, $F(x) = 1$

$F'(x) = \frac{1}{6}$, $F'(x) = \frac{2}{6} = \frac{1}{3}$, $F'(x) = \frac{3}{12} = \frac{1}{4}$, $F'(x) = 0$
M1 M1

$$f(x) = \begin{cases} \frac{1}{6} & -1 < x < 0 \\ \frac{1}{3} & 0 < x < 2 \\ \frac{1}{4} & 2 < x < \frac{8}{3} \\ 0 & \text{elsewhere} \end{cases}$$
 A1

c) $P(1 < X < 2.5) = F(2.5) - F(1) = \frac{4+3(2.5)}{12} - \frac{1+2(1)}{6}$ M1 M1

$= \frac{23}{24} - \frac{1}{2} = \frac{11}{24}$ A1

d) $E(X) = \int_{-1}^0 \frac{x}{6} dx + \int_0^2 \frac{x}{3} dx + \int_2^{8/3} \frac{x}{4} dx$

$= \frac{x^2}{12} \Big|_{-1}^0 + \frac{x^2}{6} \Big|_0^2 + \frac{x^2}{8} \Big|_2^{8/3}$ M1

$= \frac{1}{12}(0-1) + \frac{1}{6}(4-0) + \frac{1}{8}(\frac{64}{9}-4) = \frac{2}{3} - \frac{1}{12} + \frac{7}{18} = \frac{35}{36}$ M1 A1

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13) a) $\vec{R} = \begin{pmatrix} 2 \\ -3 \end{pmatrix} + \begin{pmatrix} 5 \\ 2 \end{pmatrix} + \begin{pmatrix} -2 \\ -11 \end{pmatrix} = \begin{pmatrix} 5 \\ -12 \end{pmatrix} N$ → M1 B1

$|\vec{R}| = \sqrt{5^2 + (-12)^2} = 13N$ A1

~~Alt~~ $|\vec{R}| = \sqrt{(2+5-2)^2 + (-3-11)^2} = 13N$ (M1) (M1) (A1)

b) $\begin{vmatrix} 5 & x \\ -12 & y \end{vmatrix} = \begin{vmatrix} 2 & 2 \\ -3 & 3 \end{vmatrix} + \begin{vmatrix} 5 & -2 \\ 2 & 3 \end{vmatrix} + \begin{vmatrix} -2 & 3 \\ -11 & -2 \end{vmatrix}$

$5y + 12x = (6+6) + (15+4) + (4+33)$ M1 M1 M1 M1 sum

$5y + 12x = 68$ A1

c) $y = 0$ $12x = 68$ M1
 $x = \frac{17}{3}$ OR $5\frac{2}{3}$ A1

d) $\vec{F}_4 = (-5\hat{i} + 12\hat{j}) N$ OR $\begin{pmatrix} -5 \\ 12 \end{pmatrix} N$ M1 A1 with unit

12



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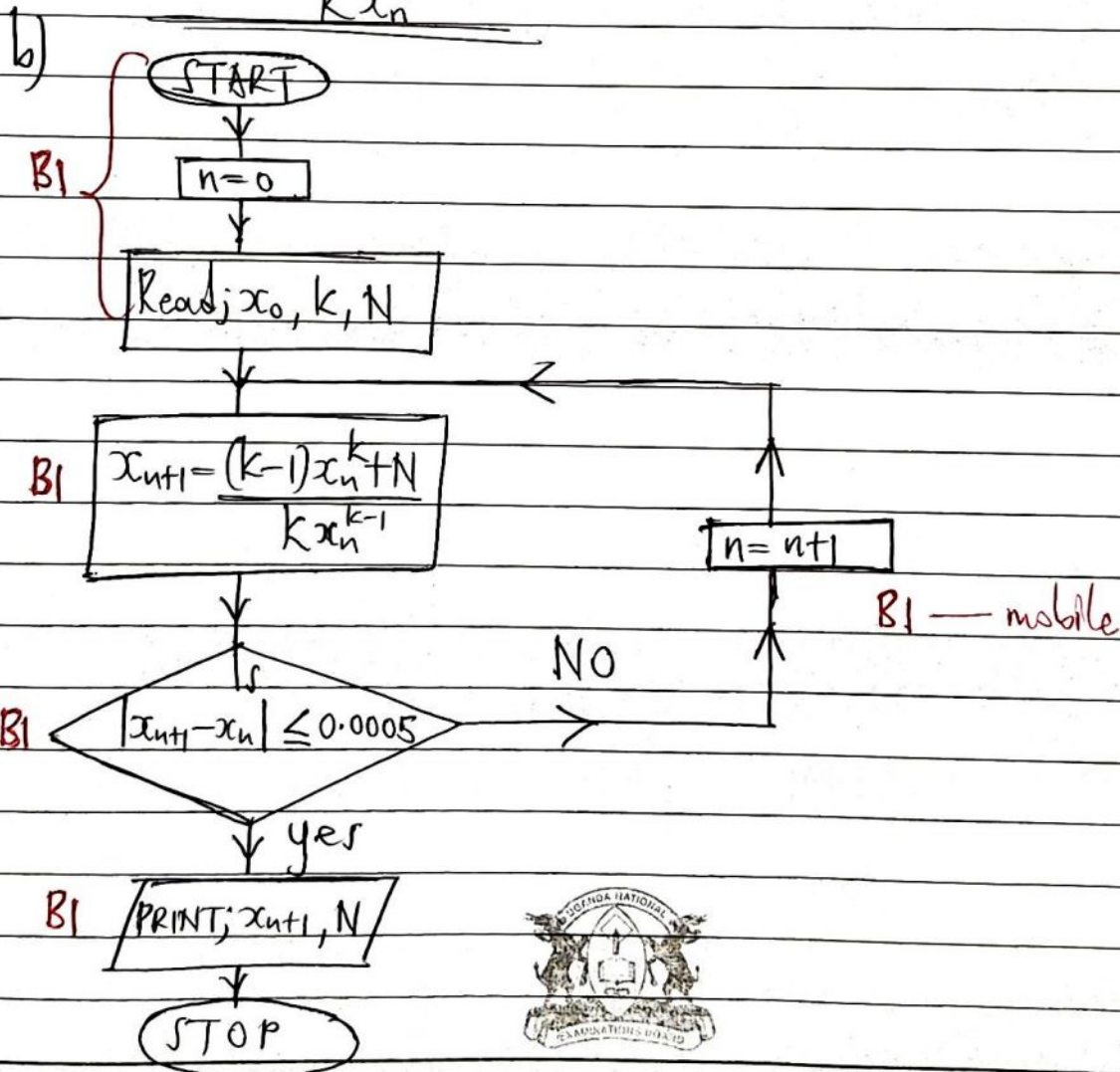
14) a) $x = \sqrt[k]{N} = N^{\frac{1}{k}}$

$\Rightarrow x^k = N \quad x^k - N = 0$

$f(x) = x^k - N$ B1, $f'(x) = kx^{k-1}$ B1

$x_{n+1} = x_n - \frac{x_n^k - N}{kx_n^{k-1}} = \frac{kx_n^k - x_n^k + N}{kx_n^{k-1}}$ M1

$x_{n+1} = \frac{(k-1)x_n^k + N}{kx_n^{k-1}}$ B1



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c)

n	x_n	x_{n+1}	$ x_{n+1} - x_n $	$ x_{n+1} - x_n \leq 0.0005$
0	1.6	1.9935	0.3935	No
1	1.9935	1.9054	0.0881	No
2	1.9054	1.8989	0.0065	No
3	1.8989	1.8989	0	Yes

B1

B1

$x_{n+1} = 1.899$ A1

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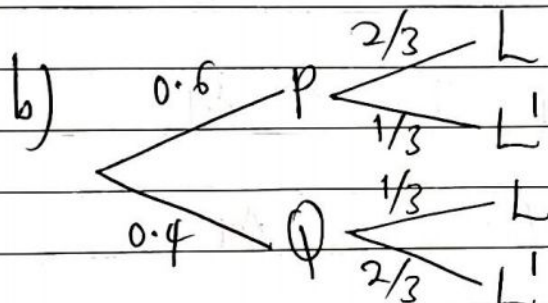
15 a) (i)
$$= \left(\frac{3 \times 1 \times 3}{10 \times 2 \times 10} \right) + \left(\frac{7 \times 1 \times 3}{10 \times 2 \times 10} \right) + \left(\frac{7 \times 1 \times 7}{10 \times 2 \times 10} \right)$$

$$= \frac{9 + 21 + 49}{200} = \frac{79}{200} = \underline{\underline{0.395}}$$
M1 - any 2 args
M1 - summing
A1

(ii)
$$= \left(\frac{3 \times 1 \times 3}{10 \times 2 \times 10} \right) + \left(\frac{3 \times 1 \times 7}{10 \times 2 \times 10} \right) + \left(\frac{7 \times 1 \times 7}{10 \times 2 \times 10} \right) + \left(\frac{3 \times 1 \times 7}{10 \times 2 \times 10} \right)$$

$$= \frac{9 + 21 + 49 + 21}{200} = \frac{100}{200} = \underline{\underline{\frac{1}{2}}}$$
M1 M1
A1

Alt
$$1 - 0.395 - (0.7 \times 0.5 \times 0.3) = \underline{\underline{\frac{1}{2}}}$$
(M1) (B1) (A1)



(i)
$$P(L) = \left(0.6 \times \frac{2}{3} \right) + \left(0.4 \times \frac{1}{3} \right) = \frac{8}{15}$$
 OR 0.5333
M1 M1 A1

(ii)
$$P(P/L') = \frac{0.6 \times \frac{1}{3}}{\left(0.6 \times \frac{1}{3} \right) + \left(0.4 \times \frac{2}{3} \right)} = \frac{\frac{1}{5}}{\frac{1}{5} + \frac{4}{15}}$$

$$= \frac{3}{7}$$
 OR 0.4286
M1 B1
A1



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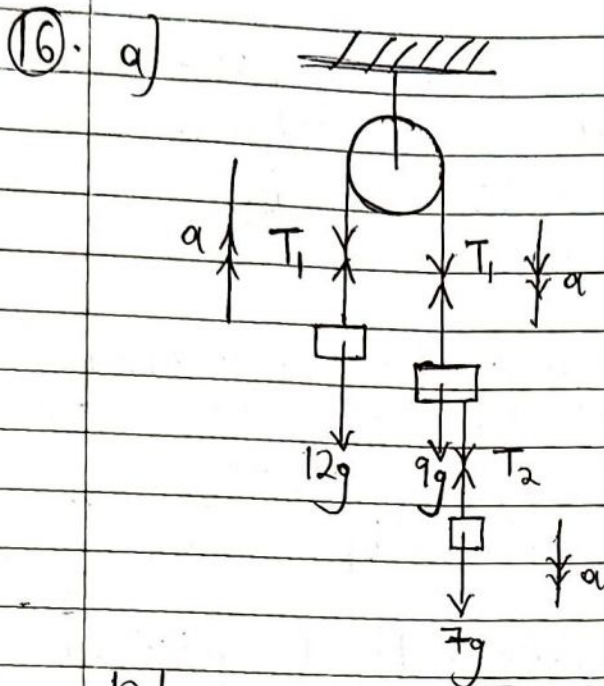
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Mubiru John
0703946946

12 kg mass

$$T_1 - 12g = 12a \quad \text{--- (1) M1}$$

9 kg mass

$$T_2 + 9g - T_1 = 9a \quad \text{--- (2) M1}$$

7 kg mass

$$7g - T_2 = 7a \quad \text{--- (3) M1}$$

(1) + (2) + (3)

$$\begin{array}{l} T_1 - 12g = 12a \\ T_2 + 9g - T_1 = 9a \\ + \quad 7g - T_2 = 7a \\ \hline 4g = 28a \end{array} \quad \text{M1}$$

(c)

$$4g = 28a$$

$$a = \frac{g}{7} = \frac{9.8}{7} = 1.4 \text{ ms}^{-2}$$

(u) $T_1 = 12(1.4) + 12(9.8) \text{ M1}$ $T_2 = 7(9.8) - 7(1.4) \text{ M1}$ A1 with units

$T_1 = 134.4 \text{ N}$ A1

$T_2 = 58.8 \text{ N}$ A1

b) $V = 0 + 1.4 \times 1.5 =$



M1 A1 with units

P425/1
PURE MATHEMATICS
Paper 1
Nov. /Dec. 2022
3 hours



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Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

Answer all the eight questions in section A and any five questions from section B.

Any additional question(s) answered will not be marked.

All necessary working must be shown clearly.

Begin each answer on a fresh sheet of paper.

Graph paper is provided.

Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

SECTION A (40 MARKS)

Answer all the questions in this section.

1. Solve the simultaneous equations:

$$2 \log_{10} y = \log_{10} 2 + \log_{10} x$$

$$2^y = 4^x \quad (05 \text{ marks})$$

2. Solve $5 \tan^2 A - 5 \tan A = 2 \sec^2 A$ for $0^\circ \leq A \leq 360^\circ$. (05 marks)

3. The position vectors of points P and Q are given by $OP = i - 2j + k$ and $OQ = 3i - 4j + 6k$ respectively. Point R divides the line PQ in the ratio $2 : -3$. Determine the coordinates of the point R . (05 marks)

4. Find the equation of the tangent to the curve $x^3 + 2y^3 + 3xy = 0$ at the point $(2, -1)$. (05 marks)

5. Solve the inequality $\frac{5 - 4x}{1 - x} < 3$ (05 marks)

6. An inverted conical container has a hole at the bottom. A liquid is dripping through the hole at a rate of $2 \text{ cm}^3 \text{ s}^{-1}$. When the depth of the liquid in the container is $x \text{ cm}$, its volume is $\frac{1}{3} \pi x^3 \text{ cm}^3$. Find the rate at which the level of the liquid is decreasing when x is 5 cm . (05 marks)

7. A line L passes through the point of intersection of the lines $x - 3y - 4 = 0$ and $y + 3x - 2 = 0$. If L is perpendicular to the line $4y + 3x = 0$, determine the equation of the line L . (05 marks)

8. Using the substitution $y = Vx$ or otherwise, solve the differential equation

$$x \frac{dy}{dx} = 2y + x$$

(05 marks)

SECTION B (60 MARKS)

Answer any five questions from this section.

All questions carry equal marks.

9. (a) Given the geometrical progression (G.P.) 2, 6, 18, 54, ... find the sum of the first ten terms of the G.P. (03 marks)
- (b) In an arithmetical progression (A.P.), the sum of the fifth and sixteenth terms is 44. The sum of the first 18 terms is three times the sum of the first ten terms. Determine the:
- (i) value of the first term.
 - (ii) common difference of the A.P.
 - (iii) sum of the first 30 terms of the A.P. (09 marks)
10. Express $\frac{11x-1}{(1-x)^2(2+3x)}$ in partial fractions.
- Hence evaluate $\int_0^{1/2} \frac{11x-1}{(1-x)^2(2+3x)} dx$ giving your answer in the form $k + \ln b$ where k is an integer and b is a fraction. (12 marks)
11. The equation of a line is $r = (\lambda - 1)i + (\lambda + 2)j + (2\lambda - 4)k$. The line is parallel to the direction vector $2i + 3j + k$.
- (a) Find the equation of the plane containing the line. (10 marks)
 - (b) Calculate the distance between the origin and the plane. (02 marks)
12. Expand $\left(\frac{1+3x}{1-x}\right)^{1/2}$ up to the term in x^3 .
- Hence substitute $x = 1/5$ to evaluate $\sqrt{8}$ correct to two decimal places. (12 marks)
13. The point $P(at^2, 2at)$ is on the parabola $y^2 = 4ax$. The chord OQ passes through the origin O . The tangent at P is parallel to the chord OQ . The tangents to the parabola at P and Q meet at a point R . Determine the coordinates of points Q and R in terms of a and t . (12 marks)
14. (a) Differentiate $\frac{(x^2+1)}{(x+1)^3}$ with respect to x . (04 marks)
- (b) Given that $x = \frac{3t}{t+3}$ and $y = \frac{4t+1}{t-2}$, find $\frac{d^2y}{dx^2}$ in terms of t in the simplest form. (08 marks)

15. (a) A chord AB subtends an angle θ radians at the centre O of a circle of radius r . The area of the circle is three times the area of the minor segment AB . Show that $3\theta = 3\sin\theta + 2\pi$. (06 marks)
- (b) Given that $\tan \alpha = \sec \alpha - \frac{1}{3}$, find the values of:
- (i) $\cos \alpha$. (06 marks)
- (ii) $\tan \alpha$. (04 marks)
16. (a) Sketch the curve $y = 5x(2 - x)$. (08 marks)
- (b) The area bounded by the curve and the x -axis is rotated about the x -axis through one revolution. Determine the volume of the solid generated. (04 marks)

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①. $2 \log_{10} y = \log_{10} 2 + \log_{10} x$

$\log_{10} y^2 = \log_{10} 2x \Rightarrow y^2 = 2x$ B1

$2^y = 4^x \quad 2^y = 2^{2x} \Rightarrow y = 2x$ B1

$4x^2 - 2x = 0 \quad 2x(2x - 1) = 0$ M1 $x = 0, \frac{1}{2}$

$\therefore \underline{x = \frac{1}{2}}$ A1 $y = 2\left(\frac{1}{2}\right) \underline{y = 1}$ A1

②. $5 \tan^2 A - 5 \tan A = 2(1 + \tan^2 A)$ M1

$3 \tan^2 A - 5 \tan A - 2 = 0$

$\tan A = \frac{5 \pm \sqrt{(-5)^2 - 4(3)(-2)}}{2(3)} = 2, -\frac{1}{3}$ M1 A1

$A = \underline{63.43^\circ, 161.57^\circ, 243.43^\circ, 341.57^\circ}$ A1 A1

③. $\underline{r = -3 \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} + 2 \begin{pmatrix} 3 \\ -4 \\ 6 \end{pmatrix}} = \begin{pmatrix} 3 \\ -6 \\ -9 \end{pmatrix} + \begin{pmatrix} -6 \\ 8 \\ -12 \end{pmatrix} = \begin{pmatrix} -3 \\ 2 \\ -9 \end{pmatrix}$ M1 M1 M1 A1

$\underline{R(-3, 2, -9)}$



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④ $3x^2 + 6y^2 \frac{dy}{dx} + 3y + 3x \frac{dy}{dx} = 0$ M1

$(2, -1) \quad 3(4) + 6(1) \frac{dy}{dx} + 3(-1) + 3(2) \frac{dy}{dx} = 0$ M1

$$\frac{dy}{dx} = -\frac{9}{12} = -\frac{3}{4}$$
 A1

$y + 1 = -\frac{3}{4}(x - 2)$ M1

$4y + 3x = 2$ A1

⑤ $\frac{5-4x}{1-x} - 3 < 0 \quad \frac{5-4x-3+3x}{1-x} < 0 \quad \frac{2-x}{1-x} < 0$ M1-M1

Critical values $x = 1, 2$ B1

	$x < 1$	$1 < x < 2$	$x > 2$	
$\frac{2-x}{1-x}$	+	-	+	M1
	<u>$1 < x < 2$</u>			A1

⑥ $V = \frac{1}{3}\pi x^3 \quad \frac{dV}{dx} = \pi x^2$ M1 $\frac{dV}{dt} = 2$

$\frac{dx}{dt} = \frac{dx}{dV} \cdot \frac{dV}{dt} = \frac{2}{\pi x^2} = \frac{2}{\pi \times 5^2} = \frac{2}{25\pi}$ M1 A1

OR $\frac{2}{25 \times \frac{22}{7}} = \frac{7}{275}$ A1

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⑦. $X - 3y - 4 = 0$ & $3X + y - 2 = 0$

$$\begin{array}{r} 3X - 9y - 12 = 0 \\ - (3X + y - 2 = 0) \quad M1 \\ \hline -10y - 10 = 0 \end{array} \quad \begin{array}{l} y = -1 \\ X = 4 + 3(-1) = 1 \end{array}$$

$(1, -1)$ A1 $4y = -3x$ $y = -\frac{3x}{4}$

L $y + 1 = \frac{4}{3}(x - 1)$ B1 M1

$3y + 3 = 4x - 4$ $3y = 4x - 7$ A1

Alt

$X - 3y - 4 + \lambda(3X + y - 2) = 0$

$(\lambda - 3)y = -(3\lambda + 1)x + 2\lambda + 4$

$\frac{3\lambda + 1}{3 - \lambda} = \frac{4}{3}$

$9\lambda + 3 = 12 - 4\lambda$

$13\lambda = 9$

$\lambda = \frac{9}{13}$

$\left(\frac{9}{13} - 3\right)y = -\left(\frac{27}{13} + 1\right)x + \frac{18}{13} + 4$

$(9 - 39)y = -(27 + 13)x + 18 + 52$

$-30y = -40x + 70$

$3y = 4x - 7$



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⑧ $y = Vx \quad \frac{dy}{dx} = V + x \frac{dV}{dx} \quad B1$

$x \left(V + x \frac{dV}{dx} \right) = 2Vx + x \quad M1$

$V + x \frac{dV}{dx} = 2V + 1 \quad x \frac{dV}{dx} = V + 1$

$\int \frac{dV}{V+1} = \int \frac{dx}{x} \quad M1 \quad \ln(V+1) + C_1 = \ln x + C_2 \quad A1$

$\ln(V+1) = \ln x + k$

$\ln(y+x) - \ln x = \ln x + k$

$\ln(y+x) = \ln x^2 k \quad y+x = kx^2 \quad A1$

Alt $\frac{dy}{dx} - \frac{2y}{x} = 1 \quad M1 \quad I.F = e^{\int \frac{-2}{x} dx} = e^{-2 \ln x} = e^{\ln(\frac{1}{x^2})}$

$I.F = \frac{1}{x^2} \quad B1$

$\frac{y}{x^2} = \int \frac{1}{x^2} dx \quad M1$

$\frac{y}{x^2} = -\frac{1}{x} + C \quad A1 \quad \frac{y}{x^2} + \frac{1}{x} = C$



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⑨. a) $a = 2, r = \frac{6}{2} = 3$ $S_{10} = \frac{2(3^{10} - 1)}{3 - 1} = 59048$
B1 m1 A1

b) $U_5 + U_{16} = 44$ $a + 4d + a + 15d = 44$
 $2a + 19d = 44$ — ① m1
 $S_{18} = 3S_{10}$

$\frac{18}{2} [2a + 17d] = 3 \times \frac{10}{2} [2a + 9d]$ m1 m1
 $18a + 153d = 30a + 135d$ m1
 $2a = 18d$ $2a = 3d$ — ② B1

$3d + 19d = 44$ $22d = 44$ $d = 2$

(i) $2a = 3(2)$ $a = 3$ A1

(ii) $d = 2$ A1

(iii) $S_{30} = \frac{30}{2} [2 \times 3 + 29 \times 2] = 960$
m1 A1



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10. $\frac{11x-1}{(1-x)^2(2+3x)} = \frac{A}{1-x} + \frac{B}{(1-x)^2} + \frac{C}{2+3x}$ M1

$$A(1-x)(2+3x) + B(2+3x) + C(1-x)^2 = 11x-1$$

$x=1$ $5B=10$ $B=2$ M1 A1

$x=-\frac{2}{3}$ $\frac{25}{9}C = -\frac{25}{3}$ $C=-3$ A1

$x=0$ $2A+2B+C=-1$
 $A = -\frac{1}{2} + \frac{3}{2} - 2$ $A=-1$ A1

$\frac{11x-1}{(1-x)^2(2+3x)} = \frac{-1}{1-x} + \frac{2}{(1-x)^2} - \frac{3}{2+3x}$ B1

$$\int_0^{1/2} \frac{11x-1}{(1-x)^2(2+3x)} dx = \int_0^{1/2} \left\{ \frac{-1}{1-x} + \frac{2}{(1-x)^2} - \frac{3}{2+3x} \right\} dx$$

$$= \ln(1-x) \Big|_0^{1/2} + \frac{2}{1-x} \Big|_0^{1/2} - \ln(2+3x) \Big|_0^{1/2}$$
M1 M1 M1 A1

$$= \ln \frac{1}{2} - \ln 1 + 2 \left(\frac{1}{1/2} - 1 \right) - \left(\ln \frac{7}{2} - \ln 2 \right)$$
M1

$$= -\ln 2 + 2 + \ln \left(\frac{2}{7} \right) + \ln 2$$

$$= 2 + \ln \left(\frac{2}{7} \right)$$
A1



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11. a) $\underline{r} = \begin{pmatrix} -1 \\ 2 \\ -4 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$ B₁ B₁ B₁

$\underline{n} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 1 & 1 & 2 \\ 2 & 3 & 1 \end{vmatrix} = \begin{pmatrix} 1-6 \\ -(1-4) \\ 3-2 \end{pmatrix} = \begin{pmatrix} -5 \\ 3 \\ 1 \end{pmatrix}$ M₁ B₁

$\begin{pmatrix} x+1 \\ y-2 \\ z+4 \end{pmatrix} \cdot \begin{pmatrix} -5 \\ 3 \\ 1 \end{pmatrix} = 0$ M₁

$-5x - 5 + 3y - 6 + z + 4 = 0$ M₁ M₁ M₁

$-5x + 3y + z = 7$ A₁

b) $\underline{r} = \frac{|-7|}{\sqrt{(-5)^2 + 3^2 + 1^2}} = \frac{7}{\sqrt{35}}$ OR $\frac{\sqrt{35}}{5}$ M₁ A₁



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12. $\left(\frac{1+3x}{1-x}\right)^{1/2} = (1+3x)^{1/2} (1-x)^{-1/2}$ B1

$$(1+3x)^{1/2} = 1 + \frac{1}{2}(3x) + \frac{1}{2}\left(-\frac{1}{2}\right)\frac{(3x)^2}{2!} + \frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\frac{(3x)^3}{3!}$$

$$= 1 + \frac{3x}{2} - \frac{9x^2}{8} + \frac{27x^3}{16}$$
M1 A1

$$(1-x)^{-1/2} = 1 - \frac{1}{2}(-x) - \frac{1}{2}\left(-\frac{3}{2}\right)\frac{(-x)^2}{2!} - \frac{1}{2}\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)\frac{(-x)^3}{3!}$$

$$= 1 + \frac{1}{2}x + \frac{3x^2}{8} + \frac{5x^3}{16}$$
M1 A1

$$\left(\frac{1+3x}{1-x}\right)^{1/2} = \left\{1 + \frac{3x}{2} - \frac{9x^2}{8} + \frac{27x^3}{16}\right\} \left\{1 + \frac{1}{2}x + \frac{3x^2}{8} + \frac{5x^3}{16}\right\}$$
M1

$$= 1 + \frac{1}{2}x + \frac{3x^2}{8} + \frac{5x^3}{16} + \frac{3x}{2} + \frac{3x^2}{4} + \frac{9x^3}{16} - \frac{9x^2}{8} - \frac{9x^3}{16} + \frac{27x^3}{16}$$
M1 M1

$$\left(\frac{1+3x}{1-x}\right)^{1/2} = 1 + 2x + 2x^3$$
A1

$$\left(\frac{1+\frac{3}{5}}{1-\frac{1}{5}}\right)^{1/2} = 1 + \frac{2}{5} + \frac{2}{125}$$
M1 M1

$$\sqrt{8} = 2\left(1 + \frac{2}{5} + \frac{2}{125}\right) \approx 2.83$$
A1

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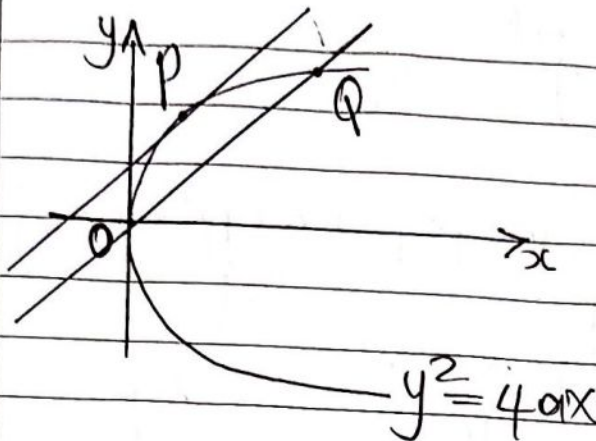
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(13)



$O(0,0)$ $P(at^2, 2at)$

Tangent at P

$$2y \frac{dy}{dx} = 4a \quad M1$$

$$\frac{dy}{dx} = \frac{2a}{y} = \frac{2a}{2at} = \frac{1}{t} \quad A1$$

$$y - 2at = \frac{1}{t}(x - at^2) \quad M1$$

$$y = \frac{x}{t} + at \quad \text{OR} \quad ty - x = at^2 \quad A1$$

Chord OQ $y - 0 = \frac{1}{t}(x - 0)$ $y = \frac{x}{t}$ or $x = ty$ M1 A1

At Q $y^2 = 4a(ty)$

$$y = 4at, \quad x = t(4at) = 4at^2 \quad M1$$

Q $(4at^2, 4at)$ A1

Tangent at Q $m = \frac{2a}{4at} = \frac{1}{2t}$

$$y - 4at = \frac{1}{2t}(x - 4at^2) \quad M1$$

$$y = \frac{x}{2t} + 2at \quad \text{OR} \quad 2ty - x = 4at^2 \quad A1$$



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$$\begin{array}{rcl} \text{At R} & | & 2ty - x = 4at^2 \\ - & | & ty - x = at^2 \quad M1 \\ \hline & & ty = 3at^2 \quad y = 3at \\ & & 3at^2 - x = at^2 \quad x = 2at^2 \end{array}$$

R(2at², 3at) A1

Alt Q(aq², 2aq)

$$M_{00} \quad \frac{2aq - 0}{aq^2 - 0} = \frac{1}{t}$$

$$\frac{2aq}{aq^2} = \frac{1}{t} \quad q = 2t$$

$$x = a(2t)^2 = 4at^2$$

$$Q(4at^2, 4at)$$

$$y = 2a(2t) = 4at$$

$$R(atq, a(t+q))$$

$$R(2at^2, 3at)$$

$$x = at(2t) = 2at^2$$

$$y = a(t+2t) = 3at$$



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14) a) $\frac{d}{dx} \left[\frac{x^2+1}{(x+1)^3} \right] = \frac{2x(x+1)^3 - 3(x+1)^2(x^2+1)}{(x+1)^6}$ M1 M1 M1

$$= \frac{(x+1)^2 [2x^2 + 2x - 3x^2 - 3]}{(x+1)^6}$$

$$= \frac{-x^2 + 2x - 3}{(x+1)^4}$$
A1

b) $x = \frac{3t}{t+3} \quad \frac{dx}{dt} = \frac{3(t+3) - 3t}{(t+3)^2} = \frac{9}{(t+3)^2}$ M1

$y = \frac{4t+1}{t-2} \quad \frac{dy}{dt} = \frac{4(t-2) - (4t+1)}{(t-2)^2} = \frac{-9}{(t-2)^2}$ M1

$\frac{dy}{dx} = \frac{-9}{(t-2)^2} \times \frac{(t+3)^2}{9} = -\frac{(t+3)^2}{(t-2)^2}$ M1 A1

$\frac{d^2y}{dx^2} = - \frac{\{2(t+3)(t-2)^2 - 2(t-2)(t+3)^2\}}{(t-2)^4} \times \frac{(t+3)^2}{9}$ M1 M1

$$= -\frac{2(t+3)(t-2)[t-2-t-3](t+3)^2}{9(t-2)^4}$$
M1



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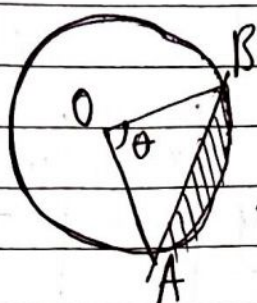
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$$\frac{d^2y}{dx^2} = \frac{-2(-5)(t+3)^3}{9(t-2)^3} = \frac{10(t+3)^3}{9(t-2)^3} \quad \text{A7}$$

(15) a)



$$\pi r^2 = 3 \left[\frac{1}{2} r^2 \theta - \frac{1}{2} r^2 \sin \theta \right]$$

$$\pi = \frac{3}{2} (\theta - \sin \theta) \quad \text{B1 B1 B1 M1}$$

$$2\pi = 3\theta - 3\sin \theta \quad \text{M1}$$

$$3\theta = 2\pi + 3\sin \theta \quad \text{B1}$$

b) $\tan \alpha = \sec \alpha - \frac{1}{3}$

$$\tan^2 \alpha = \sec^2 \alpha - \frac{2}{3} \sec \alpha + \frac{1}{9} \quad \text{M1}$$

$$1 + \tan^2 \alpha - \frac{2}{3} \sec \alpha + \frac{1}{9} = \tan^2 \alpha \quad \text{M1}$$

$$\frac{2}{3} \sec \alpha = \frac{10}{9} \quad \sec \alpha = \frac{5}{3} \quad \text{M1}$$

(i) $\cos \alpha = \frac{3}{5} \quad \text{A7}$

(ii) $\tan \alpha = \frac{5}{3} - \frac{1}{3} = \frac{4}{3} \quad \text{M1 A7}$



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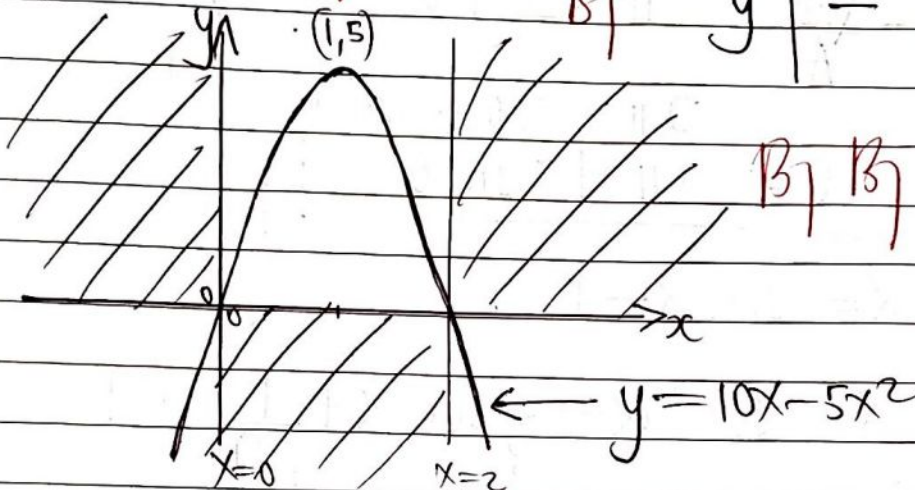
16. a) $y = 5x(2-x) = 10x - 5x^2$

$y(0) = 0$ $(0,0)$ B_1 $x(0) = 0, 2$ $(2,0)$ B_1

$\frac{dy}{dx} = 10 - 10x = 0$ M_1 $x = 1$, $y(1) = 5$ $(1,5)$ A_1

$\frac{d^2y}{dx^2} = -10 \Rightarrow (1,5)$ M_1 B_1

$x < 0$	$0 < x < 2$	$x > 2$
y	$-$	$+$



b) $V = \pi \int_0^2 (10x - 5x^2)^2 dx = \pi \int_0^2 (100x^2 - 100x^3 + 25x^4) dx$ M_1

$= \pi \left(\frac{100x^3}{3} - 25x^4 + 5x^5 \right) \Big|_0^2$ M_1

$= \pi \left(\frac{800}{3} - 400 + 160 \right) = \frac{80}{3} \pi$ OR $26\frac{2}{3} \pi$



OR $\frac{1760}{21}$ A_1

P425/2
APPLIED MATHEMATICS
Paper 2
Nov. /Dec. 2022
3 hours



UGANDA NATIONAL EXAMINATIONS BOARD

Uganda Advanced Certificate of Education

APPLIED MATHEMATICS

Paper 2

3 hours

INSTRUCTIONS TO CANDIDATES:

*Answer **all** the **eight** questions in section A and any **five** questions from section B.*

*Any additional question(s) answered will **not** be marked.*

*All necessary working **must** be shown clearly.*

Begin each answer on a fresh sheet of paper.

Graph paper is provided.

Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

In numerical work, take acceleration due to gravity g , to be 9.8 ms^{-2} .

SECTION A (40 MARKS)

Answer all the questions in this section.

1. A rough plane is inclined at an angle of $\sin^{-1}(3/5)$ to the horizontal. A car of mass 1500 kg moves up the plane with a steady speed. The coefficient of friction between the car and the plane is $1/4$. Calculate the driving force exerted by the engine of the car. (05 marks)
2. The table below shows the number of people in each of the 100 cars that passed through a check point on a certain day.

Number of people	1	2	3	4	5
Number of cars	41	33	18	6	2

Find:

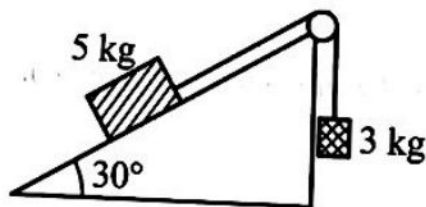
- (a) the mean number of people in each car. (02 marks)
- (b) the variance of the data. (03 marks)
3. Use the trapezium rule with seven (7) ordinates to estimate

$$\int_0^2 \frac{1}{3+4x^2} dx$$

correct to three decimal places.

(05 marks)

4. The diagram below shows a mass of 5 kg lying on a rough plane inclined at 30° to the horizontal. From the 5 kg mass, a light inextensible string passes up the line of greatest slope and over a smooth fixed pulley carrying a mass of 3 kg hanging freely. The tension in the string is just sufficient to prevent the 5 kg mass from sliding down the slope.



Determine the coefficient of friction between the 5 kg mass and the rough plane. (05 marks)

5. Two events A and B are such that $P(A) = \frac{1}{2}$, $P(B) = \frac{7}{12}$ and $P(A' \cap B) = \frac{1}{2}$. Find $P(B' \cap A)$. (05 marks)

6. The table below shows the amount of Euros equivalent to US dollars.

US dollars	79	97	105
Euros	64	78	85

Using linear interpolation/extrapolation, calculate the amount of:

- (a) dollars equivalent to 92 Euros. (03 marks)
 (b) Euros equivalent to 85 dollars. (02 marks)
7. A river which is 50 m wide flows with a speed of 3 ms^{-1} . A boat must be set to cross the river to a point directly opposite the starting point. The speed of the boat relative to the river is 4 ms^{-1} . Determine the;
 (a) velocity with which the boat sets off to cross the river. (03 marks)
 (b) time taken by the boat to cross the river. (02 marks)
8. Bag A contains 7 red balls and 4 blue balls. Bag B contains 5 red balls and 8 blue balls. A ball is randomly transferred from A to B . A ball is then randomly removed from bag B . Find the probability that:
 (a) a red ball was removed from bag B . (03 marks)
 (b) the transferred ball is blue given that a red ball is removed from B . (02 marks)

SECTION B (60 MARKS)

Answer any five questions from this section.

All questions carry equal marks.

9. The table below shows the marks of eight students (A to H) in two tests.

Student	A	B	C	D	E	F	G	H
Test 1	67	73	50	75	42	44	75	40
Test 2	82	70	64	78	52	64	80	64

- (a) (i) Plot a scatter diagram for the given data.
 (ii) Draw a line of best fit on the scatter diagram.
 (iii) Use your scatter diagram to obtain a mark in Test 2 which corresponds to a mark of 63 in Test 1. (06 marks)

- (b) (i) Calculate the rank correlation coefficient for the data.
 (ii) Comment on your result in b (i). (06 marks)

10. A particle of mass 4 kg is initially at rest at the point $A(2, -2, 3)$ metres. The particle is acted upon by a force of $F = \begin{pmatrix} 4t \\ t^2 \\ 5 \end{pmatrix}$ Newtons, where t is time in seconds. Find the;

- (a) acceleration at time t . (02 marks)
 (b) speed of the particle after 3 seconds. (05 marks)
 (c) displacement of the particle after 3 seconds. (05 marks)

11. (a) Two numbers x and y are approximated by X and Y with errors Δx and Δy respectively. Show that the relative error in approximating $\frac{x}{y}$ by $\frac{X}{Y}$ is $\left| \frac{\Delta x}{X} \right| + \left| \frac{\Delta y}{Y} \right|$ (06 marks)

(b) The values in the expression $T = \frac{673.16}{40.345}$ are rounded off correct to the given decimal places. Determine the interval within which the exact value of T can be expected to lie. (06 marks)

12. A discrete random variable X has a probability distribution given by

$$f(x) = \begin{cases} kx^2, & x = 1, 2, 3 \\ k(7-x)^2, & x = 4, 5, 6 \\ 0, & \text{elsewhere} \end{cases}$$

Find the:

- (a) (i) value of the constant k .
 (ii) mean of X .
 (iii) variance of X . (09 marks)
 (b) (i) Determine the cumulative distribution function, $F(x)$. (01 mark)
 (ii) Sketch the graph of $F(x)$. (02 marks)

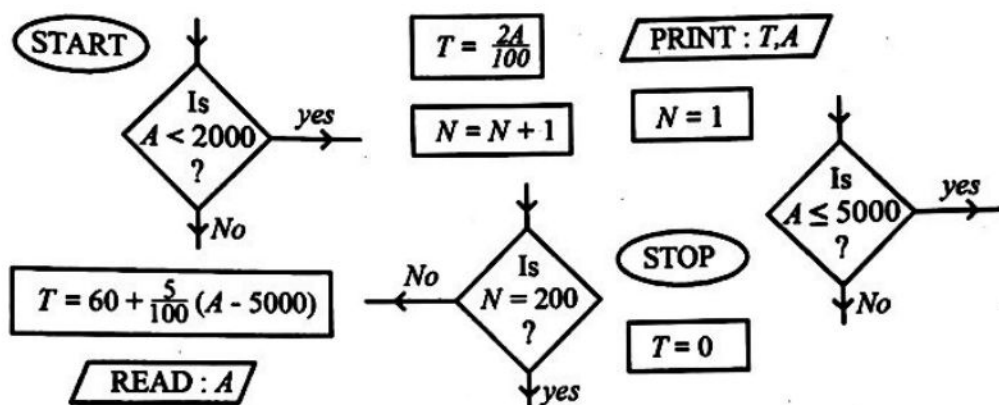
13. Forces of magnitude 2N, 2N, 3N, 4N, $2\sqrt{2}$ N and $\sqrt{2}$ N act along sides $\overline{AB}$, $\overline{BC}$, $\overline{DC}$, $\overline{AD}$, $\overline{AC}$ and $\overline{DB}$ respectively. Where $ABCD$ is a square of side 2 m. Find the:

- (a) resultant force. (09 marks)
- (b) equation of the line of action of the resultant force. (03 marks)

14. The taxation system in a company is such that the tax T based on the amount A earned falls in three categories as shown in the table below:

AMOUNT EARNED A (DOLLARS)	TAX PAID T (DOLLARS)
$A < 2000$	$T = 0$
$2000 \leq A \leq 5000$	$T = \frac{2A}{100}$
$A > 5000$	$T = 60 + \frac{5}{100}(A - 5000)$

Parts of a flow chart for computing the tax are given below.



- (a) (i) Arrange the given parts to form a complete logical flow chart. (08 marks)
- (ii) State the purpose of the flow chart. (01 mark)
- (b) Perform a dry run for the flow chart and complete the table below: (03 marks)

N	A	T
1	1500	—
2	3500	—
3	9000	—

15. The masses of packets of tea leaves are normally distributed with mean mass 600 g and standard deviation 20 g.
- (a) If a packet of tea leaves is chosen at random, find the mass exceeded by 7% of the packets. (05 marks)
- (b) If 1,000 packets of tea leaves were sold, find the number of packets that weighed less than 545 g. (07 marks)

16. Two points A and B are on the same horizontal ground. A particle P is projected from point A towards point B with an initial speed of 49 ms^{-1} at an angle of 30° to the horizontal. Two seconds later, another particle Q is projected from point B towards A with the same speed and at the same angle of elevation as P .

If the particles collide at a height h m above the ground, determine the:

- (a) value of h . (06 marks)
- (b) distance between the points A and B . (06 marks)

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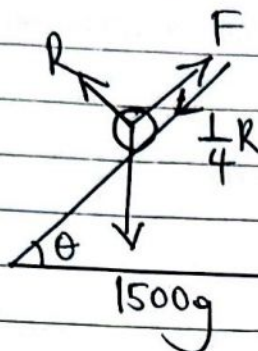
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①.



$$\theta = \sin^{-1}\left(\frac{3}{5}\right) = \cos^{-1}\left(\frac{4}{5}\right)$$

$$\sin \theta = \frac{3}{5}, \cos \theta = \frac{4}{5}, \alpha = 0$$

$$R = 1500g \cos \theta$$

$$F - 1500g \sin \theta - \frac{1}{4}R = 0$$

$$F = \left(1500 \times 9.8 \times \frac{3}{5}\right) + \left(\frac{1}{4} \times 1500 \times 9.8 \times \frac{4}{5}\right)$$

$$F = 11760 \text{ N}$$

②.

x	1	2	3	4	5	
f	41	33	18	6	2	$\Sigma f = 100$
xf	41	66	54	24	10	$\Sigma xf = 195$
x ² f	41	132	162	96	50	$\Sigma x^2 f = 481$

$$a) \bar{x} = \frac{195}{100} = 1.95$$

$$b) s^2 = \frac{481}{100} - (1.95)^2 = 1.0075$$



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③ $y = \frac{1}{3+4x^2}$, $h = \frac{2-0}{7-1} = \frac{2}{6} = \frac{1}{3}$ B1

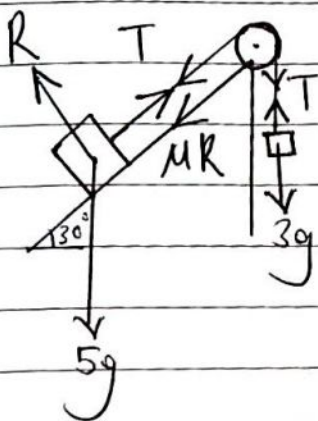
x	0	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{5}{3}$	2	Sum
	$\frac{1}{3}$						$\frac{1}{19}$	$\frac{22}{57}$
y		$\frac{9}{31}$	$\frac{9}{43}$	$\frac{1}{7}$	$\frac{9}{91}$	$\frac{9}{127}$		0.8123

$\int_0^2 \frac{1}{3+4x^2} dx = \frac{1}{2} \times \frac{1}{3} \left[\frac{22}{57} + 2 \times 0.8123 \right]$ M1

0.335 A1

05

④



$T = 3g$, $R = 5g \cos 30^\circ$

$T = 5g \sin 30^\circ + \mu R$ B1

$\Rightarrow 3g = 5g \sin 30^\circ + \mu (5g \cos 30^\circ)$ M1

$\mu = \frac{3 - 5 \sin 30^\circ}{5 \cos 30^\circ}$ M1

$\mu = 0.1155$ or $\frac{\sqrt{3}}{15}$ A1

05



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⑤ $P(B) = P(A \cap B) + P(A' \cap B)$
 $\frac{7}{12} = P(A \cap B) + \frac{1}{2}$ B1 M1

$P(A \cap B) = \frac{1}{12}$ A1

$P(A \cap B') = \frac{1}{2} - \frac{1}{12} = \frac{5}{12}$ M1 A1

05

⑥ a)

97	105	D
78	85	92

B1

$\frac{D - 105}{92 - 85} = \frac{105 - 97}{85 - 78}$ M1

$D = 113$ A1

b)

79	85	97
64	E	78

$\frac{E - 64}{85 - 79} = \frac{78 - 64}{97 - 79}$ M1

$E = \frac{206}{3}$ OR $68.667 \approx \underline{\underline{69}}$ A1

05



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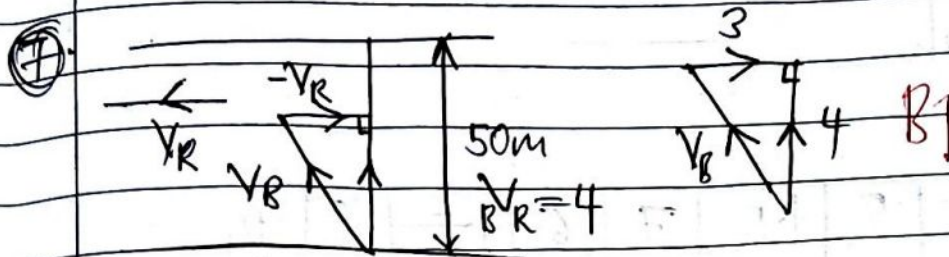
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a) $V_B = \sqrt{3^2 + 4^2} = \underline{5 \text{ ms}^{-1}}$ B1 M1 A1

b) $t = \frac{50}{4} = \underline{12.5 \text{ s}}$ M1 A1

05

8.	Bag A	<table><tr><td>7R</td></tr><tr><td>4B</td></tr></table>	7R	4B	Bag B	<table><tr><td>5R</td></tr><tr><td>8B</td></tr></table>	5R	8B
7R								
4B								
5R								
8B								

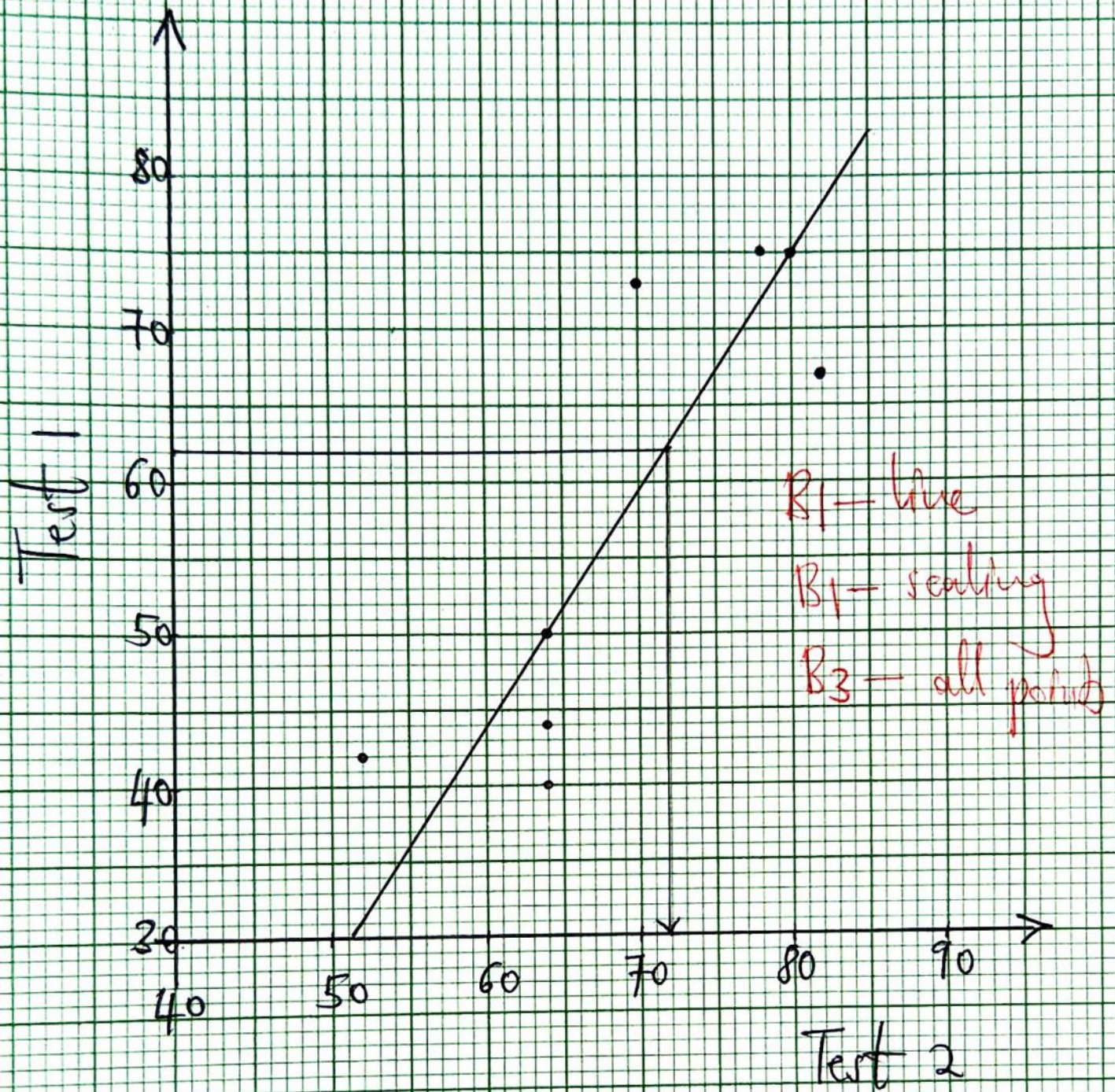
a) $P(R) = P(R_1 R_2) + P(B_1 R_2)$
 $= \left(\frac{7}{11} \times \frac{6}{14} \right) + \left(\frac{4}{11} \times \frac{5}{14} \right) = \underline{\frac{31}{77}}$ B1 M1 A1

b) $P(B_1 / R) = \frac{P(B_1 R_2)}{P(R)}$
 $= \frac{\frac{4}{11} \times \frac{5}{14}}{\frac{31}{77}} = \underline{\frac{10}{31}}$ M1 A1

05



9. a) i) ii) iii)



$\overline{\text{Test 2}} = 72 \pm 5$ A1

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9. a) See graph

b)

r_1	4	3	5	1.5	7	6	1.5	8	B_1
r_2	1	4	6	3	8	6	2	6	B_1
d^2	9	1	1	2.25	1	0	0.25	4	

$$\sum d^2 = 18.5 \quad B_1$$

$$r = 1 - \frac{6 \times 18.5}{8^3 - 8} = 0.7798 \quad M_1 \quad A_1$$

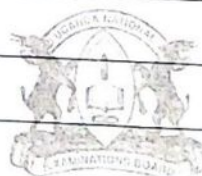
$$\approx \underline{\underline{0.78}}$$

OR — significant at 5% B_1

OR — insignificant at 1%

OR — high positive correlation

(12)



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10. a) $\underline{F} = m \underline{a}$ $\begin{pmatrix} 4t \\ t^2 \\ 5 \end{pmatrix} = 4 \underline{a}$ $\underline{a} = \begin{pmatrix} t \\ t^2/4 \\ 5/4 \end{pmatrix} \text{ m s}^{-2}$

b) $\underline{V} = \int \begin{pmatrix} t \\ t^2/4 \\ 5/4 \end{pmatrix} dt = \begin{pmatrix} t^2/2 \\ t^3/12 \\ 5t/4 \end{pmatrix} + \underline{C}_1$ $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + \underline{C}_1$ $\underline{C}_1 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$\underline{V} = \begin{pmatrix} t^2/2 \\ t^3/12 \\ 5t/4 \end{pmatrix} \text{ m s}^{-1}$

$\underline{V}(3) = \begin{pmatrix} 4.5 \\ 2.25 \\ 3.75 \end{pmatrix} \text{ m s}^{-1}$

Speed = $\sqrt{(4.5)^2 + (2.25)^2 + (3.75)^2}$
 $= 6.27495 \text{ m s}^{-1}$

c) $\underline{r} = \int \begin{pmatrix} t^2/2 \\ t^3/12 \\ 5t/4 \end{pmatrix} dt = \begin{pmatrix} t^3/6 \\ t^4/48 \\ 5t^2/8 \end{pmatrix} + \underline{C}_2$

$\begin{pmatrix} 2 \\ -2 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + \underline{C}_2$ $\underline{C}_2 = \begin{pmatrix} 2 \\ -2 \\ 3 \end{pmatrix}$ $\underline{r} = \begin{pmatrix} t^3/6 + 2 \\ t^4/48 - 2 \\ 5t^2/8 + 3 \end{pmatrix}$

Displacement $\underline{r}(3) = \begin{pmatrix} 4.5 \\ 1.6875 \\ 5.625 \end{pmatrix} \text{ m}$ OR $\begin{pmatrix} 9/2 \\ 27/16 \\ 45/8 \end{pmatrix} \text{ m}$

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$$\textcircled{11}. a) e = \frac{x}{y} - \frac{X}{Y} = \frac{(X+\Delta x) \cdot (Y-\Delta y)}{(Y+\Delta y)(Y-\Delta y)} - \frac{X}{Y} \quad M1$$

$$e = \frac{XY - X\Delta y + Y\Delta x - \Delta x\Delta y}{Y^2 - (\Delta y)^2} - \frac{X}{Y} \quad M1$$

Since Δx and Δy are so small, $\Delta x\Delta y \approx 0$, $(\Delta y)^2 \approx 0$ B1

$$e = \frac{X}{Y} + \frac{Y\Delta x - X\Delta y}{Y^2} - \frac{X}{Y}$$

$$e = \frac{Y\Delta x - X\Delta y}{Y^2}$$

$$R.E = \left| \frac{Y\Delta x - X\Delta y}{Y^2} \right| = \left| \frac{Y\Delta x - X\Delta y}{XY} \right| \leq \left| \frac{Y\Delta x}{XY} \right| + \left| \frac{X\Delta y}{XY} \right|$$

M1 B1

$$R.E_{\max} = \left| \frac{\Delta x}{X} \right| + \left| \frac{\Delta y}{Y} \right| \quad B1$$

$$b) T_{\max} = \frac{673.165}{40.3445} = 16.6854 \quad M1 A1$$

$$T_{\min} = \frac{673.155}{40.3455} = 16.6848 \quad B1 M1 A1$$

$$[16.6848, 16.6854] \quad A1$$



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12. a)

x	1	2	3	4	5	6
$P(X=x)$	k	$4k$	$9k$	$9k$	$4k$	k

(i) $k + 4k + 9k + 9k + 4k + k = 1$ B1 M1
 $28k = 1$ A1
 $k = \frac{1}{28}$

(ii) $E(X) = k + 8k + 27k + 36k + 20k + 6k$ B1 M1
 $= 98k$
 $= 98 \times \frac{1}{28} = \frac{7}{2}$ or 3.5 A1

(iii) $E(X^2) = k + 16k + 81k + 144k + 100k + 36k$
 $= 378k$
 $= 378 \times \frac{1}{28} = \frac{27}{2}$ or 13.5 B1

$Var(X) = \frac{27}{2} - \left(\frac{7}{2}\right)^2 = 1.25$ or $\frac{5}{4}$ M1 A1

b)

x	1	2	3	4	5	6
$FG(x)$	k	$5k$	$14k$	$23k$	$27k$	$28k$
$F(x)$	$\frac{1}{28}$	$\frac{5}{28}$	$\frac{14}{28}$	$\frac{23}{28}$	$\frac{27}{28}$	1

B1


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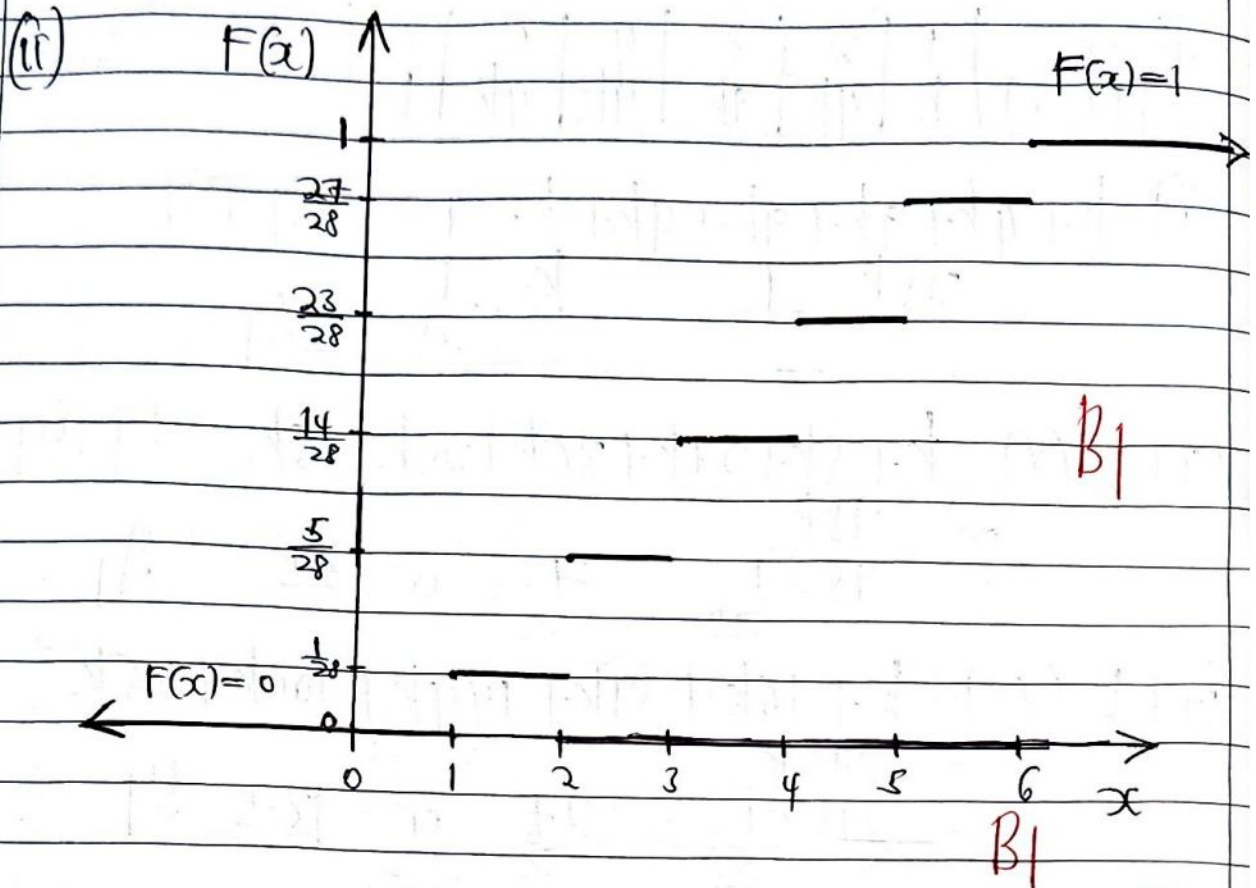
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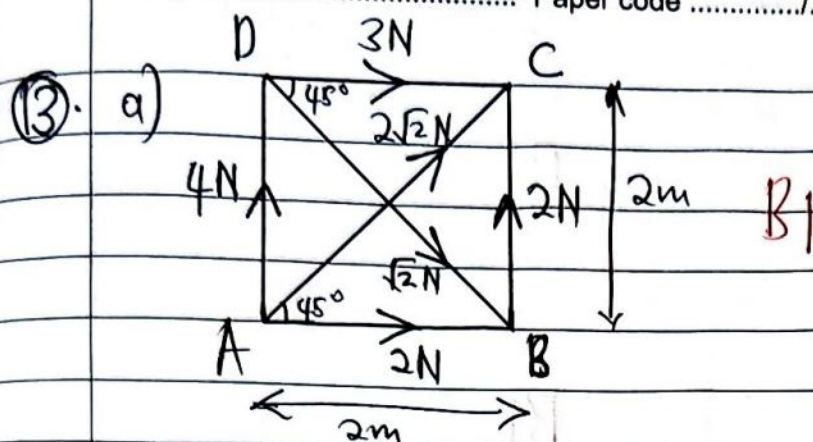
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$$\begin{pmatrix} X \\ Y \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 2 \end{pmatrix} + \begin{pmatrix} 3 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 4 \end{pmatrix} + \begin{pmatrix} 2\sqrt{2} \cos 45^\circ \\ 2\sqrt{2} \sin 45^\circ \end{pmatrix} + \begin{pmatrix} \sqrt{2} \cos 45^\circ \\ -\sqrt{2} \sin 45^\circ \end{pmatrix} \quad \begin{matrix} M \\ M \end{matrix}$$

$$\begin{pmatrix} X \\ Y \end{pmatrix} = \begin{pmatrix} 8 \\ 7 \end{pmatrix} \quad \begin{matrix} B1 \\ B1 \end{matrix} \quad R = \sqrt{8^2 + 7^2} = \sqrt{113} \text{ N OR } 10.6301 \text{ N} \quad \begin{matrix} M \\ A \end{matrix}$$

$$\alpha = \tan^{-1}\left(\frac{7}{8}\right) = \sin^{-1}\left(\frac{7}{\sqrt{113}}\right) = \cos^{-1}\left(\frac{8}{\sqrt{113}}\right) = 41.19^\circ \quad A$$

$$\sqrt{113} \text{ N due } E 41.19^\circ N \text{ OR } N 48.81^\circ E \quad B1$$

b)

$$G = (2 \times 2) - (3 \times 2) - (\sqrt{2} \times 2 \cos 45^\circ) = -4 \text{ Nm} \quad B1$$

$$-4 = \begin{vmatrix} x & 8 \\ y & 7 \end{vmatrix} \quad M$$

$$-4 = 7x - 8y$$



$$8y = 7x + 4$$

12

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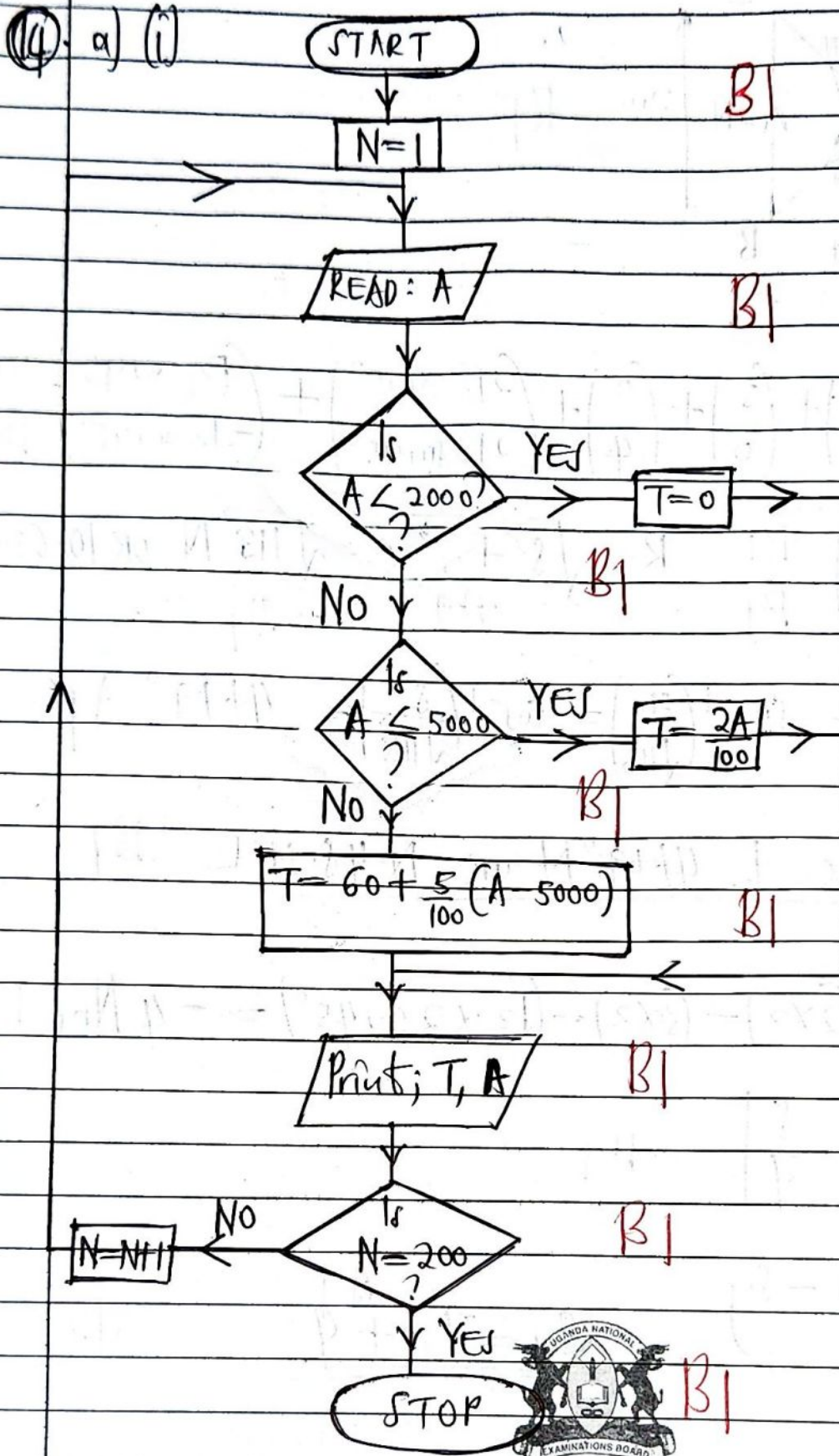
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(ii) Computing and printing taxes. B1

b)

N	A	T
1	1500	0
2	3500	70
3	900	260

B1
B1
B1

(12)



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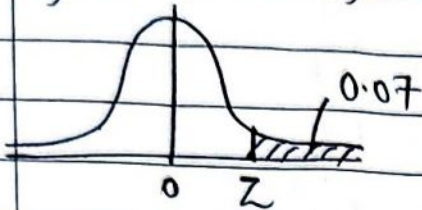
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(15). $\mu = 600g, \sigma = 20g$

a) $P(X > a) = \frac{7}{100} = 0.07$ B1



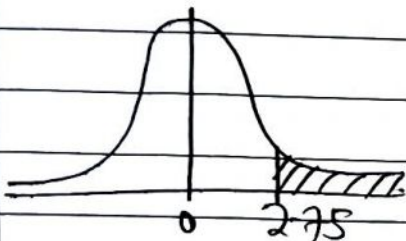
$\frac{a - 600}{20} = 1.476$ B1 M1 M1

$a = 629.52g$ A1

b) $P(X < 545) = P\left(Z < \frac{545 - 600}{20}\right)$ M1

$= P(Z < -2.75)$ B1

$= P(Z > 2.75)$



$= 0.5 - 0.4970$ M1 B1

$= 0.003$ A1

No of packets $= 1000 \times 0.003$ M1
 $= 3$ packets A1

(12)



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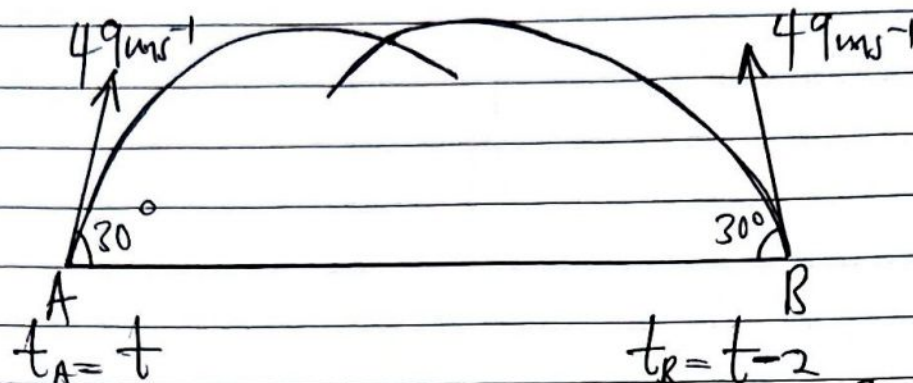
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(16) a)



$$r_A = \begin{pmatrix} 49t \cos 30^\circ \\ 49t \sin 30^\circ - \frac{1}{2}gt^2 \end{pmatrix} \quad r_B = \begin{pmatrix} 49(t-2) \cos 30^\circ \\ 49(t-2) \sin 30^\circ - \frac{1}{2}g(t-2)^2 \end{pmatrix}$$

$$49t \sin 30^\circ - \frac{1}{2}gt^2 = 49(t-2) \sin 30^\circ - \frac{1}{2}g(t-2)^2$$

$$-\frac{1}{2}gt^2 = -49 - \frac{1}{2}gt^2 + 19.6t - 19.6$$

$$19.6t = 68.6$$

$$t = 3.5s$$

Alt $t_A = t+2, t_B = t \Rightarrow t = 1.5s$

$$h = 49 \times 2.5 \sin 30^\circ - \frac{1}{2} \times 9.8 \times 3.5^2 = 25.725m$$

b) $AB = 49 \times 3.5 \cos 30^\circ + 49 \times 1.5 \cos 30^\circ$

$$= 212.1762m$$



P425/1
PURE MATHEMATICS
Paper 1
Nov. / Dec. 2020
3 hours



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Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

Answer all the eight questions in section A and any five questions from section B.

Any additional question(s) answered will not be marked.

All necessary working must be shown clearly.

Begin each answer on a fresh sheet of paper.

Graph paper is provided.

Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

SECTION A: (40 MARKS)

Answer all the questions in this section.

1. Solve the equation: $\sin x + \sin 2x + \sin 3x = 0$ for $0^\circ \leq x \leq 180^\circ$. (05 marks)
2. (a) Express $Z = \frac{3+i}{1-i}$ in the form $a + bi$, where a and b are integers. (03 marks)
(b) Find the argument of Z . (02 marks)
3. Given that $y = \ln \left\{ x \sqrt{(x+1)^3} \right\}$, find $\frac{dy}{dx}$. (05 marks)
4. A plane is perpendicular to the vector $r = (i + 3j - 2k)$ and contains the point $P(-2, 0, 4)$. Determine the equation of the plane. (05 marks)
5. Evaluate $\int_0^{\pi/3} \tan^2 \frac{1}{2}x \, dx$. (05 marks)
6. In how many ways can the letters of the word BUNDESLIGA be arranged if;
(a) there is no restriction? (02 marks)
(b) the vowels must be together? (03 marks)
7. (a) Show that the curve whose parametric equations are $x = 9 \cos \theta$ and $y = 12 \sin \theta$ represents an ellipse. (03 marks)
(b) Determine the eccentricity of the ellipse. (02 marks)
8. Find the gradient of the curve $x^2 \tan x - xy - 2y^2 = -2$ at the point $(0,1)$. (05 marks)

SECTION B: (60 MARKS)

Answer any five questions from this section. All questions carry equal marks.

9. (a) A polynomial $P(x)$ is given by $P(x) = (x+2)(x-1)Q(x) + (ax+b)$ where $Q(x)$ is the quotient and $ax+b$ is the remainder. When $P(x)$ is divided by $x-1$, the remainder is 4 and when it is divided by $x+2$, the remainder is 1. Find the values of a and b . (05 marks)

- (b) (i) Expand $(1 + x^4)^{-1/2}$ up to the fourth term.
 (ii) Use the first two terms of the expansion to find the value of

$$\frac{1}{\sqrt{144.0144}}$$

correct to two significant figures.

(07 marks)

10. A circle passes through the points (1, 3), (2, 2) and (5, 7).

Find the equation of the;

(a) circle.

(07 marks)

(b) tangent to the circle at the point (1, 3).

(05 marks)

11. Express $\frac{2-x+x^2}{(1+x)(1-x)^2}$ in partial fractions.

Hence evaluate $\int_0^{1/2} \frac{(2-x+x^2)}{(1+x)(1-x)^2} dx$ correct to three decimal places.

(12 marks)

12. (a) Determine the angle between the vectors

$$p = i + 9j + 4k \text{ and } q = i - j + 2k.$$

(05 marks)

(b) The vector equations of two lines are $r_1 = i - 3j + 4k + \lambda(-i - 3j + k)$
 and $r_2 = -2j + 5k + \mu(i + 2j - k)$.

Find the coordinates of the point of intersection of the two lines.

(07 marks)

13. Express $8 \sin \theta - 15 \cos \theta$ in the form $R \sin(\theta - \alpha)$ where R and α are constants.

Hence find the maximum value of $8 \sin \theta - 15 \cos \theta$ and the smallest positive value of θ at which the maximum occurs.

(12 marks)

14. (a) Solve the simultaneous equations:

$$2x^2 - 5xy + 2y^2 = 0$$

$$x + y = 6$$

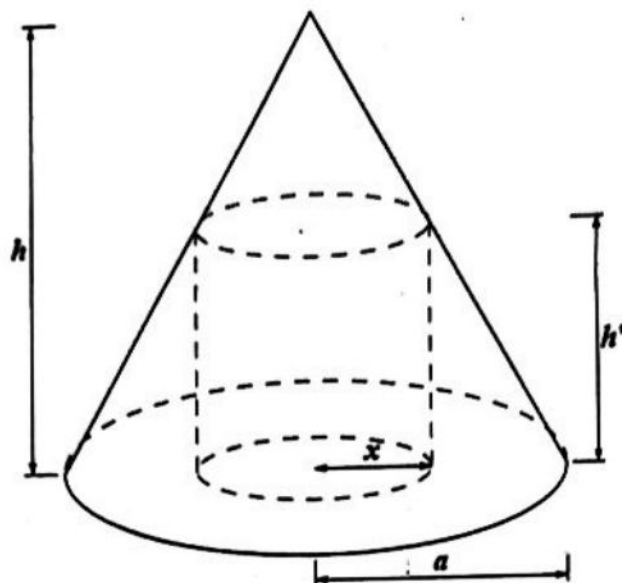
(05 marks)

(b) If α and β are the roots of the equation $x^2 - px + q = 0$, find the equation whose roots are $\frac{\alpha^2 - 1}{\alpha}$ and $\frac{\beta^2 - 1}{\beta}$.

(07 marks)

Turn Over

15. The diagram below shows a cone of radius a and height h , in which a cylinder of radius x is inscribed.



- (a) Express the height h' of the cylinder in terms of x , a and h . (04 marks)
- (b) Show that $V' = \frac{4}{9}V$ where V' is the greatest volume of the cylinder that can be inscribed in the given cone of volume V . (08 marks)
16. (a) Solve the differential equation
- $$\frac{ds}{dt} = \frac{2e^{2t}}{\sqrt{S}} \quad \text{given that } S = 9 \text{ when } t = 0.$$
- (b) Determine the value of;
- (i) t when $S = 16$
 - (ii) S when $t = 2.4$
- (Give your answer to 2 significant figures) (12 marks)

END

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Mubito Johnson

1. $\sin x + \tan x + \sec x = 0$ for $0^\circ \leq x \leq 180^\circ$.

$2\sin x \cos x + \sin x = 0$.

$\sin x (2\cos x + 1) = 0$.

$\sin x = 0$ or $\cos x = -1/2$.

for $\cos x = -1/2$, $x = \cos^{-1}(-1/2) = \{120^\circ, 240^\circ, \dots\}$

for $\sin x = 0$, $2x = \sin^{-1} 0 = \{0, 180^\circ, 360^\circ, \dots\}$

$x = \{0^\circ, 90^\circ, 180^\circ, \dots\}$

Hence $x = \{0^\circ, 90^\circ, 180^\circ, 120^\circ\}$.

M₁ factor formulae.

M₁ factorization.

A₁ for both $\sin x = 0$ & $\cos x = -1/2$.

B₁ for $\sin^{-1} 0 = \{0, 180^\circ\}$

B₁ for $x = \cos^{-1}(-1/2)$

Ans: $2\sin \frac{x}{2} \cos \frac{x}{2} + 2\sin \frac{x}{2} \cos \frac{x}{2} = 0$

$2\sin \frac{x}{2} [\cos \frac{x}{2} + \cos \frac{x}{2}] = 0$

$2\sin \frac{x}{2} [2\cos \frac{x}{2}] = 0$

either $\sin \frac{x}{2} = 0$, $\cos x = 0$, $\cos \frac{x}{2} = 0$.

Ans: $\sin x + 2\sin x \cos x + 3\sin x - 4\sin^3 x = 0$

$4\sin x + 2\sin x \cos x - 4\sin^3 x = 0$

$2\sin x [2 + \cos x - 2\sin^2 x] = 0$

$\sin x = 0$ or $2 + \cos x - 2(1 - \cos^2 x) = 0$

$\cos x (1 + 2\cos x) = 0$

either $\sin x = 0$, $\cos x = 0$, $\cos x = -1/2$.

$x = 0^\circ, 180^\circ, 90^\circ$

$x = 120^\circ$

M₁ expressing in single angle

M₁ factorization.

A₁ for $\sin x = 0$

$\cos x = 0$

$\cos x = -1/2$

B₁

B₁

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$$2. (a) \quad z = \frac{3+i}{1-i} = \frac{(3+i)(1+i)}{(1-i)(1+i)} = \frac{3+3i+i-1}{1+1} \\ = \frac{2+4i}{2} = 1+2i$$

M₁ ^{→ Radising} M₁ ^{→ expansion}

$$(b) \quad \arg z = \tan^{-1}(2,1) \\ = 63.43^\circ$$

A₁ M₁ $\tan^{-1}(2,1)$ (accept 63.4°)

Alt: $z = \frac{3+i}{1-i}$

$$(b) \quad |z| = \frac{\sqrt{9+1}}{\sqrt{1+1}} = \frac{\sqrt{10}}{\sqrt{2}} = \sqrt{5}$$

$$\arg z = \arg(3+i) - \arg(1-i) \\ = \tan^{-1}(1/3) - \tan^{-1}(-1) \\ = 18.43^\circ - (-45^\circ) \\ = 63.43^\circ$$

M₁ A₁

$$(a) \quad z = |z| [\cos(\arg z) + i \sin(\arg z)] \\ = \sqrt{5} [\cos 63.43^\circ + i \sin 63.43^\circ] \\ = 1.00 + 1.9999i \approx 1+2i$$

^{→ modulus used} ^{→ polar form} M₁ M₁ A₁

$$3. \quad y = \ln [x \sqrt{(x+1)^2}] \\ e^y = x(x+1)^{3/2}$$

M₁ (index form / notation)

$$e^y \frac{dy}{dx} = x \cdot \frac{3}{2}(x+1)^{1/2} + (x+1)^{3/2}$$

M₁ M₁ L.H.S diff. R.H.S diff.

$$e^y \frac{dy}{dx} = (x+1)^{1/2} \left[\frac{3}{2}x + (x+1) \right]$$

$$e^y \frac{dy}{dx} = \frac{(x+1)^{1/2} (3x+2x+2)}{2}$$

$$\frac{dy}{dx} = \frac{\frac{1}{2}(x+1)^{1/2} (5x+2)}{x(x+1)^{3/2}} = \frac{5x+2}{2x(x+1)}$$

^{→ 1/2} ^{→ dx} ^{→ correct}

M₁ A₁

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Ans 1.

$$3. y = \ln \{x(x+1)^{3/2}\} = \ln x + \ln (x+1)^{3/2}$$

$$= \ln x + \frac{3}{2} \ln (x+1)$$

M₁ { log. rule }
M₁

$$\frac{dy}{dx} = \frac{1}{x} + \frac{3}{2(x+1)}$$

M₁ M₁
for $\frac{1}{x}$ (M₁) for $\frac{3}{2} \ln (x+1)$

$$= \frac{2(x+1) + 3x}{2x(x+1)}$$

$$= \frac{5x+2}{2x(x+1)}$$

A₁

Ans 2:

$$y = \ln \{x(x+1)^{3/2}\}$$

$$\frac{dy}{dx} = \frac{(x+1)^{3/2} + \frac{3}{2}x(x+1)^{1/2}}{x(x+1)^{3/2}}$$

M₁ M₁ M₁ M₁
u dv rule v dv rule product rule

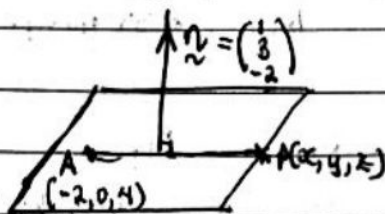
$$= \frac{(x+1)^{1/2} \{ (x+1)(2) + 3x \}}{2x(x+1)^{3/2}}$$

Ans

$$\frac{dy}{dx} = \frac{5x+2}{2x(x+1)}$$

A₁

4.



$$\vec{PA} \cdot \vec{n} = 0$$

$$\begin{pmatrix} x+2 \\ y-0 \\ z-4 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix} = 0$$

normal vector, n
B₁ M₁ M₁

$$(x+2) + 3(y-0) - 2(z-4) = 0$$

$$x + 3y - 2z + 2 + 8 = 0$$

$$x + 3y - 2z + 10 = 0$$

M₁ simplification
A₁

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4.

$$\vec{n} = \begin{pmatrix} 4 \\ 3 \\ -2 \end{pmatrix}$$

$$\vec{a} = \begin{pmatrix} -2 \\ 0 \\ 4 \end{pmatrix}, \quad \vec{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

B₁ for normal vector

$$\vec{n} \cdot \vec{n} = \vec{n} \cdot \vec{a}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 3 \\ -2 \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} -2 \\ 0 \\ 4 \end{pmatrix}$$

$$x + 3y - 2z = -2 + 0 - 8$$

$$x + 3y - 2z = -10$$

M₁ M₁ for a·n

M₁ simplification
A₁

5.

$$\int_0^{\pi/3} \tan^2 x_2 dx$$

$$d/dx (\tan x) = \sec^2 x, \quad 1 + \tan^2 x = \sec^2 x$$

$$\int_0^{\pi/3} (\sec^2 x_2 - 1) dx = \int_0^{\pi/3} \sec^2 x_2 dx - \int_0^{\pi/3} 1 dx$$

B₁ for $\tan^2 x_2 = \sec^2 x_2 - 1$

$$= 2 \tan x_2 - x \Big|_0^{\pi/3}$$

M₁ correct integration
A₁ correct integral

$$= (2 \tan \pi/6 - \pi/3) - (2 \tan 0 - 0)$$

M₁ limit values

$$= \frac{2}{\sqrt{3}} - \pi/3 \quad (\text{accept } 0.107502987) \quad A_1$$

G. BUNDRESHWA

a) 10! = 3,628,800. M₁ A₁

b) BUNDLES (URIA)

$$7! \times 4!$$

$$m_1 \quad m_1$$

$$5040 \times 24$$

$$= 120,960$$

A₁

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ALT:

5.

$$\int_0^{\pi/3} \tan^2 \frac{x}{2} dx$$

Let $t = \tan \frac{x}{2}$, $\frac{dt}{dx} = \sec^2 \frac{x}{2}$, $dx = \frac{2dt}{(1+t^2)}$

x	t
0	0
$\pi/3$	$1/\sqrt{3}$

B₁ for d
\$ \Delta
Limit

$$\int_0^{\pi/3} \tan^2 \frac{x}{2} dx = \int_0^{1/\sqrt{3}} \frac{2t^2}{(1+t^2)} dt$$

$$= \int_0^{1/\sqrt{3}} \left(2 - \frac{2}{1+t^2} \right) dt$$

$$= 2t - 2 \tan^{-1} t \Big|_0^{1/\sqrt{3}}$$

M₁ A₁

$$= \frac{2}{\sqrt{3}} - \frac{2\pi}{6}$$

M₁ Substn.

$$= \frac{2}{\sqrt{3}} - \frac{\pi}{3} \text{ or } 0.1075$$

A₁

7(a) $x = 9 \cos \theta$, $y = 12 \sin \theta$

$\cos \theta = x/9$, $\sin \theta = y/12$

B₁ for sin & cos
Substn.

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\frac{y^2}{144} + \frac{x^2}{81} = 1 \Rightarrow \frac{x^2}{81} + \frac{y^2}{144} = 1$$

M₁ θ elimination
A₁

But $a^2 = b^2(1-e^2)$

M₁ Substn.

$$81 = 144(1-e^2)$$

$$e^2 = 1 - 9/16 = 7/16$$

$$e = \frac{\sqrt{7}}{4}$$

A₁ correct +ve e-value.

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8. $x^2 \tan x - xy - 2y^2 = -2$ at $P(0, 1)$.

$$2x \tan x + x^2 \sec^2 x - y - x \frac{dy}{dx} - 4y \frac{dy}{dx} = 0$$

M_1 A_1

$$2(0) \tan(0) + 0^2 \sec^2 0 - 1 - (1) \frac{dy}{dx} - 4(1) \frac{dy}{dx} = 0$$

M_1

Subst. $x=0, y=1$

$$-1 - 4 \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{1}{4}$$

M_1 A_1

ALT:

$$2x \tan x + x^2 \sec^2 x - y - x \frac{dy}{dx} - 4y \frac{dy}{dx} = 0$$

M_1 A_1

$$(x + 4y) \frac{dy}{dx} = 2x \tan x + x^2 \sec^2 x - y$$

$$\frac{dy}{dx} = \frac{2x \tan x + x^2 \sec^2 x - y}{x + 4y}$$

M_1 A_1

At $P(0, 1)$, $\frac{dy}{dx} = \frac{0 + 0 - 1}{0 + 4}$

M_1 Subst. $x=0, y=1$

$$= -\frac{1}{4}$$

A_1

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9(a) $P(x) = (x+2)(x-1)Q(x) + (ax+b)$

when $Q(x) = x-1=0$, $x=1$, $P(1) = a+b=4 \Rightarrow a+b=4$ --- (i) B₁

when $Q(x) = x+2=0$, $x=-2$, $P(2) = -2a+b=1 \Rightarrow -2a+b=1$ --- (ii) B₁

Subtract (i) - (ii) gives

$$3a = 3$$

$$a = 1$$

M₁ solving (i)

A₁

when $a=1$, $b = 4-a = 4-1$
 $= 3$

A₁

(b) (i) $(1+x^4)^{-1/2} = 1 + \frac{-1/2}{1!}(x^4)^1 + \frac{(-1/2)(-3/2)}{2!}(x^4)^2 + \frac{(-1/2)(-3/2)(-5/2)}{3!}(x^4)^3 + \dots$ M₁

$$= 1 - \frac{1}{2}x^4 + \frac{3}{8}x^8 - \frac{15}{128}x^{12} + \dots$$

M₁ simplification

$$= 1 - \frac{1}{2}x^4 + \frac{3}{8}x^8 - \frac{5}{16}x^{12} + \dots$$

A₁

(ii) $(1+x^4)^{-1/2} \approx 1 - \frac{1}{2}x^4$

$$(1+x^4)^{-1/2} \approx 1 - \frac{1}{2}x^4 \approx \frac{1}{(1+x^4)^{1/2}} = \frac{1}{\sqrt{(144.0144)}} = \frac{1}{\sqrt{(144(1+0.0001))}}$$

$$= \frac{1}{12\sqrt{(1+0.0001)}} = \frac{1}{12\sqrt{(1+(1/10)^4)}} = \frac{1}{12(1+(1/10)^4)^{1/2}}$$

$$\approx \frac{1}{12} \left(1 - \frac{1}{2}(1/10)^4\right)$$

M₁

$$\approx 0.08332$$

A₁

$$\approx 0.083 \text{ (2 s.f.)}$$

B₁

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10(a) $(x-a)^2 + (y-b)^2 = r^2$ for radius r , centre (a,b)

At $(5,7)$, $(5-a)^2 + (7-b)^2 = r^2$ --- (i)

At $(1,3)$, $(1-a)^2 + (3-b)^2 = r^2$ --- (ii)

At $(2,2)$, $(2-a)^2 + (2-b)^2 = r^2$ --- (iii)

Substn (ii)-(i) gives, $a+b=8$ --- (iv) B₁

(ii) Substn (iii)-(i) gives, $-a+b=1$ --- (v) B₁

Add (iv)+(v) gives, $2b=9$ M₁

$$b = 9/2$$

$$\therefore a = 8 - b = 8 - 9/2 = 7/2$$

Centre $(7/2, 9/2)$

Hence radius r is given by;

$$r^2 = (5 - 7/2)^2 + (7 - 9/2)^2$$

$$r = \sqrt{\frac{34}{4}} = \frac{\sqrt{34}}{2} \text{ units.}$$

Circle equation is, $(x - 7/2)^2 + (y - 9/2)^2 = \frac{34}{4}$ M₁

$$x^2 + y^2 - 7x - 9y + 24 = 0$$

A₁

for
 $x = 1$

(b) Gradient of the radius = $\frac{3 - 9/2}{1 - 7/2}$ M₁

$$= 3/5$$

A₁

Gradient of tangent = $-5/3$ B₁ tangent grad.

$$\therefore \frac{y-3}{x-1} = -5/3$$

M₁

$$3y = 5x + 14$$

A₁

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Q1. Using the general eqn, $x^2 + y^2 + 2gx + 2fy + C = 0$.

At A(1,3) ; $1 + 9 + 2g + 6f + C = 0$

$\therefore 2g + 6f + C = -10$ (1) B₁

At B(2,2) ; $4 + 4 + 4g + 4f + C = 0$

$4g + 4f + C = -8$ (2) B₁

At C(5,7) ; $25 + 49 + 10g + 14f + C = 0$

$10g + 14f + C = -74$ (3) B₁

Subtract (2)-(1) ; $2g - 2f = 2$

$g - f = 1$ (4)

Subtract (3)-(2) ; $6g + 10f = -66$

$3g + 5f = -33$ (5)

$\therefore 3(1+f) + 5f = -33$

$3 + 8f = -33$

$f = -9/4$

$g = -7/2$

$C = 24$

A₁ (for b, g, C)

Hence the equation of the circle is ; $x^2 + y^2 - 7x - 9y + 24 = 0$ B₁

(b) $2x + 2y \frac{dy}{dx} - 7 - 9 \frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{7 - 2x}{2y - 9}$

At A(1,3) ; $\frac{dy}{dx} = \frac{7 - 2}{6 - 9} = -5/3$

$\frac{y - 3}{x - 1} = -5/3$

$3y + 5x = 14$

M₁ diff.

M₁ diff. Subst.

A₁

M₁

A₁

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11. $\frac{2-x+x^2}{(1+x)(1-x)^2} = \frac{A}{(1+x)} + \frac{B}{(1-x)} + \frac{C}{(1-x)^2}$ B₁

$$2-x+x^2 = A(1-x)^2 + B(1+x)(1-x) + C(1-x)$$

for $x=1$; $2=2C$ M₁
 $C=1$ A₁

for $x=-1$; $4=4A$ M₁
 $A=1$ A₁

for $x=0$; $2=A+B+C$ M₁
 $2=1+B+1$ A₁
 $B=0$ A₁

Hence $\frac{2-x+x^2}{(1+x)(1-x)^2} = \frac{1}{1+x} + \frac{1}{(1-x)^2}$ B₁

$$\therefore \int_0^{1/2} \frac{2-x+x^2}{(1+x)(1-x)^2} dx = \int_0^{1/2} \left(\frac{1}{1+x} + \frac{1}{(1-x)^2} \right) dx$$
 M₁

$$= \ln(1+x) \Big|_0^{1/2} + \frac{1}{(1-x)} \Big|_0^{1/2}$$
 A₁ A₁

$$= (\ln \frac{3}{2} - \ln 1) + \left(\frac{1}{1-\frac{1}{2}} - 1 \right)$$
 M₁

$$= \ln \frac{3}{2} - \ln 1 + 2 - 1$$

$$= \ln \frac{3}{2} + 1$$

$$= 1.405$$
 A₁

{ A₁ for @
correct integ.
value }

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12 (a) $\vec{p} = \hat{i} + 9\hat{j} + 4\hat{k} = \begin{pmatrix} 1 \\ 9 \\ 4 \end{pmatrix}$, $\vec{q} = \hat{i} - \hat{j} + 2\hat{k} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$

$$\begin{pmatrix} 1 \\ 9 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} = \sqrt{1^2 + 9^2 + 4^2} \cdot \sqrt{1^2 + (-1)^2 + 2^2} \cos \lambda$$

$$1 - 9 + 8 = \sqrt{98} \sqrt{6} \cos \lambda$$

$$\cos \lambda = \frac{0}{\sqrt{98} \sqrt{6}}$$

$$\lambda = \cos^{-1} 0$$

$$\lambda = 90^\circ$$

(b) $\vec{r}_1 = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ -3 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ -3 \\ 1 \end{pmatrix} = \begin{pmatrix} 1-\lambda \\ -3-3\lambda \\ 4+\lambda \end{pmatrix}$

$$\vec{r}_2 = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ -2 \\ 5 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} = \begin{pmatrix} \mu \\ -2+2\mu \\ 5-\mu \end{pmatrix}$$

$$\therefore \begin{pmatrix} 1-\lambda \\ -3-3\lambda \\ 4+\lambda \end{pmatrix} = \begin{pmatrix} \mu \\ -2+2\mu \\ 5-\mu \end{pmatrix}$$

At the point of intersection:

$$1-\lambda = \mu$$

$$-3-3\lambda = -2+2\mu$$

$$4+\lambda = 5-\mu$$

Solving (i) and (ii) simultaneously:

$$4+\lambda = 5 - \frac{-3-3\lambda+2}{2}$$

$$\lambda = -3, \mu = 1-3 = 4$$

$$\vec{r}_1 = \begin{pmatrix} 1-\lambda \\ -3-3\lambda \\ 4+\lambda \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \\ 1 \end{pmatrix}$$

Point is (4, 6, 1).

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Ans: $\vec{p} \cdot \vec{q} = \begin{pmatrix} 1 \\ 9 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} = 1 - 9 + 8 = 0$

M₁ M₁

$$\cos \theta = \frac{0}{|\vec{p}| |\vec{q}|} = \frac{0}{7\sqrt{12}}$$

M₁

$$\therefore \cos \theta = 0$$

$$\theta = \cos^{-1} 0$$

$$\theta = 90^\circ$$

M₁

A₁

Ans: $\cos \theta = \frac{\vec{p} \cdot \vec{q}}{|\vec{p}| |\vec{q}|}$

$$= \frac{\begin{pmatrix} 1 \\ 9 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}}{7\sqrt{12}}$$

dot prod.
modulus (both)
equating

M₁

M₁

M₁

$$\cos \theta = 0$$

$$\theta = \cos^{-1} 0$$

$$\theta = 90^\circ$$

M₁

A₁

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13. $8 \sin \theta - 15 \cos \theta = R \sin(\theta - \alpha) = R \sin \theta \cos \alpha - R \cos \theta \sin \alpha$

$R \sin \alpha = 15$

B_1

$R \cos \alpha = 8$

B_1

$\frac{R \sin \alpha}{R \cos \alpha} = \frac{15}{8}$

M_1

$\tan \alpha = 15/8$

$\alpha = \tan^{-1} 15/8 \longrightarrow M_1$

$\alpha = 61.93^\circ$

A_1

$R^2 \cos^2 \alpha + R^2 \sin^2 \alpha = 8^2 + 15^2$

M_1

$R^2 = 289$

$R = 17$

A_1

$\therefore 8 \sin \theta - 15 \cos \theta = 17 \sin(\theta - 61.93^\circ)$

B_1

Max. value is 17

A_1

It occurs when $\sin(\theta - 61.93^\circ) = 1$

M_1

$\theta - 61.93^\circ = 90^\circ$

M_1

$\theta = 151.93^\circ$

A_1

Alt: Harmonic form.

$8 \sin \theta - 15 \cos \theta = R \sin(\theta - \alpha) = a \sin \theta - b \cos \theta$

$\alpha = \tan^{-1}(15/8)$

B_1, B_1, M_1, M_1, A_1

$= 61.93^\circ$

$R = \sqrt{15^2 + 8^2}$

M_1

$= 17$

A_1

$\therefore 8 \sin \theta - 15 \cos \theta = 17 \sin(\theta - 61.93^\circ)$

B_1

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14(a) $2x^2 - 5xy + 2y^2 = 0$ (i)
 $x + y = 6$ (ii)
 M_1 solve $2x^2 - 5x(6-x) + 2(6-x)^2 = 0$
 $2x^2 - 30x + 5x^2 + 2(36 - 12x + x^2) = 0$
 $9x^2 - 54x + 72 = 0$ or $x^2 - 6x + 8 = 0$
 B_1 solving $(x-4)(x-2) = 0$
 M_1 solving $x = 4$ or $x = 2$
 A_1 (x-value) $x = 4$ or $x = 2$
 A_1 (y-value) $y = 2$ or $y = 4$

$2(b-y)^2 - 5y(b-y) + 2y^2 = 0$
 $72 - 24y + 2y^2 - 5by + 5y^2 + 2y^2 = 0$
 $9y^2 - 54y + 72 = 0$
 $y = \frac{54 \pm \sqrt{54^2 - 4(9)(72)}}{2 \times 9}$
 $y = 4$ or $y = 2$
 $x = 2$ or $x = 4$

14(b) $x^2 - px + q = x^2 - (\alpha + \beta)x + \alpha\beta = 0$

B_1 solve $\therefore \alpha + \beta = p$ (i)
 $\alpha\beta = q$ (ii)

New roots: $\frac{\alpha^2 - 1}{\alpha}$, $\frac{\beta^2 - 1}{\beta}$

Sum of new roots: $\left(\frac{\alpha^2 - 1}{\alpha}\right) + \left(\frac{\beta^2 - 1}{\beta}\right)$

M_1 (common denominator) $= \frac{(\alpha^2 - 1)\beta + \alpha(\beta^2 - 1)}{\alpha\beta} = \frac{(\alpha + \beta)(\alpha\beta - 1)}{\alpha\beta}$

A_1 $= \frac{qp - p}{q}$

Product of new roots: $\left(\frac{\alpha^2 - 1}{\alpha}\right) \cdot \left(\frac{\beta^2 - 1}{\beta}\right)$

M_1 (common denominator) $= \frac{\alpha^2\beta^2 - \alpha^2 - \beta^2 + 1}{\alpha\beta} = \frac{(\alpha\beta)^2 - (\alpha + \beta)^2 + 2\alpha\beta + 1}{\alpha\beta}$

A_1 $= \frac{(q^2 - p^2 + 2q + 1)}{q}$

M_1, A_1 New equation is: $x^2 - \left(\frac{qp - p}{q}\right)x + \left(\frac{q^2 - p^2 + 2q + 1}{q}\right) = 0$

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Ans 14(e)

$$2(x^2 + y^2) - 5xy = 0$$

$$2\{(x+y)^2 - 2xy\} - 5xy = 0$$

$$2\{6^2 - 2xy\} - 5xy = 0$$

$$72 = 9xy$$

$$xy = 8$$

$$y = 8/x$$

$$\therefore x + \frac{8}{x} = 6$$

$$x^2 - 6x + 8 = 0$$

$$(x-4)(x-2) = 0$$

$$\therefore x = 4, x = 2$$

$$y = 2, y = 4$$

B₁

M₁

A₁

A₁

Ans

14(f)

$$2x^2 - 5xy + 2y^2 = 0 \quad \dots (i)$$

$$x + y = 6 \quad \dots (ii)$$

Taking $y = mx$, Subst. in (i)

$$2(x^2 + y^2) - 5xy = 0$$

$$2(x^2 + m^2x^2) - 5x^2m = 0$$

$$x^2(2 + 2m^2 - 5m) = 0$$

$$\text{either } x = 0 \text{ or } 2m^2 - 5m + 2 = 0$$

$$(2m-1)(m-2) = 0$$

$$m = \frac{1}{2} \text{ or } m = 2$$

Subst. $y = mx$ in (ii) $x + y = 6 \Rightarrow x + mx = 6, x(1+m) = 6$

When $m = \frac{1}{2}, x = \frac{6}{(1+\frac{1}{2})} = \frac{6x}{(\frac{3}{2})} = \frac{6 \times 2}{3} = 4, y = 4(\frac{1}{2}) = 2. \quad A_1$

When $m = 2, x = \frac{6}{(1+2)} = \frac{6}{3} = 2, y = 2(2) = 4. \quad A_1$

B₁ for quadratic in m
M₁ solving

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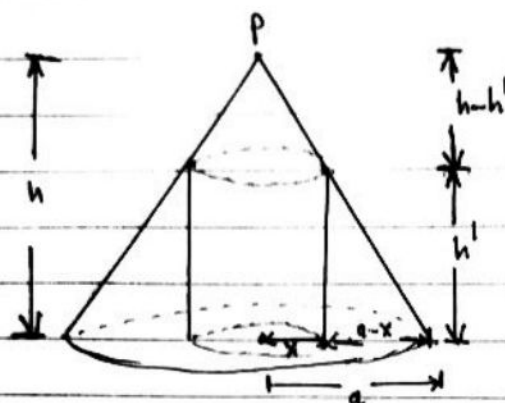
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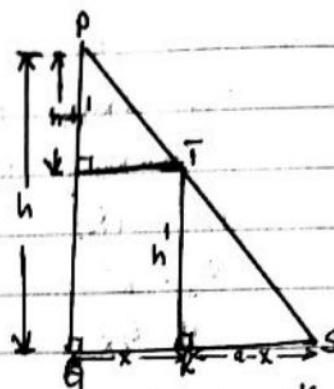
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Using similar Δ 's: $\frac{h'}{a-x} = \frac{h}{a}$
 $\therefore h' = \frac{h(a-x)}{a}$



Identifying similar Δ 's
 B_1, M_1, m_1
 A_1 in terms of h, a, x .
 L.H.S ratio
 R.H.S ratio

$$V = \frac{4}{3}\pi a^2 h$$

$$V' = \pi x^2 h'$$

$$V' = \pi x^2 \left(\frac{h(a-x)}{a} \right)$$

m_1

$$V' = \frac{\pi h}{a} (ax^2 - x^3)$$

m_1

$$\frac{dV'}{dx} = \frac{\pi h}{a} (2ax - 3x^2)$$

m_1

For V'_{max} , $\frac{dV'}{dx} = 0$ i.e. $\frac{\pi h}{a} (2ax - 3x^2) = 0$

m_1

$\therefore x=0$ or $x = \frac{2a}{3}$ But $x > 0$.

$x = \frac{2}{3}a$

A_1

$$V'_{max} = \frac{\pi h}{a} \left(\frac{4a^3}{9} - \frac{8a^3}{27} \right)$$

M_1 Subst. $x = \frac{2a}{3}$

$$= \frac{4}{9}a^3 \frac{\pi h}{a} \left(1 - \frac{2}{3} \right)$$

A_1 for V'_{max}

$$V' = \frac{4}{9} \left(\frac{1}{3} \pi a^2 h \right)$$

B_1

$$V' = \frac{4}{9} V$$

16 (a) $\frac{ds}{dt} = \frac{2e^{2t}}{\sqrt{s}}$, $s=9$, $t=0$.

$$\sqrt{s} ds = 2e^{2t} dt.$$

$$\int s^{1/2} ds = 2 \int e^{2t} dt.$$

$$\frac{2}{3} s^{3/2} + C_1 = 2 \cdot e^{2t} \cdot \frac{1}{2} + C_2.$$

$$\frac{2}{3} s^{3/2} = e^{2t} + K, \quad K = C_2 - C_1.$$

$$\frac{2}{3} (9)^{3/2} = e^{2(0)} + K.$$

$$18 = 1 + K.$$

$$K = 17.$$

$$\therefore \frac{2}{3} s^{3/2} = e^{2t} + 17.$$

M₁ Separating Variables.

M₁ Integrating.

A₁.

M₁

B₁

A₁

16 (b) (i) $t=?$, $s=16$.

$$\frac{2}{3} (16)^{3/2} = e^{2t} + 17.$$

$$\frac{128}{3} - 17 = e^{2t}.$$

$$2t = \ln\left(\frac{77}{3}\right)$$

$$t = \frac{1}{2} \ln\left(\frac{77}{3}\right)$$

$$t = 1.6226.$$

$$t \approx 1.6 \text{ (2 s.f.)}$$

M₁ Substn.

M₁ for $t = \frac{1}{2} \ln\left(\frac{77}{3}\right)$

A₁

(ii) $s=?$, $t=2.4$.

$$\frac{2}{3} s^{3/2} = e^{4.8} + 17.$$

$$s^{3/2} = \frac{3}{2} (138.5104)$$

$$s = \left(\frac{3}{2} (138.5104)\right)^{2/3} = 85.079.$$

M₁ Substn.

M₁ s-substn $s=85$. A₁.

$\frac{4}{27} \pi^2$

P425/2
APPLIED MATHEMATICS
Paper 2
Nov./ Dec. 2020
3 hours



UGANDA NATIONAL EXAMINATIONS BOARD
Uganda Advanced Certificate of Education
APPLIED MATHEMATICS

Paper 2
3 hours

INSTRUCTIONS TO CANDIDATES:

Answer all the eight questions in section A and any five questions from section B.

Any additional question(s) answered will not be marked.

All necessary working must be shown clearly.

Begin each answer on a fresh sheet of paper.

Graph paper is provided.

Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

In numerical work, take acceleration due to gravity g , to be 9.8 ms^{-2} .

SECTION A: (40 MARKS)

Answer all the questions in this section.

1. The position vector of a particle at any time t seconds is given by
$$\mathbf{r}(t) = (t^2 + 4t)\mathbf{i} + (3t - t^3)\mathbf{j} \text{ metres.}$$

Calculate the speed of the particle when $t = 3$ seconds. (05 marks)

2. The table below shows the values of x and the corresponding values of a function $f(x)$.

x	0.3	0.6	0.9	1.2
$f(x)$	3.00	3.22	3.69	4.06

Use linear interpolation to find;

- (a) $f(x)$ when $x = 0.4$ (03 marks)
(b) x when $f(x) = 3.82$ (02 marks)
3. The table below shows the price indices of beans, maize, rice and meat with the corresponding weights.

Item	Price index 2008 (2007 = 100%)	Weight
Beans	105	4
Maize	x	7
Rice	104	2
Meat	113	5

Calculate the;

- (a) value of x given that the price indices of maize in 2007 and 2008 using 2006 as the base year are 112 and 130 respectively. (02 marks)
(b) weighted price index for 2008 using 2007 as the base year. (03 marks)
4. A particle moves with simple harmonic motion (SHM) about a mean position O with periodic time of $\frac{2\pi}{3}$ seconds. When the particle is 0.8 m from one extreme end, its speed is 3.6 ms^{-1} . Determine the amplitude of the motion. (05 marks)
5. The numbers $X = 1.2$, $Y = 1.33$ and $Z = 2.245$ have been rounded off to the given decimal places. Find the maximum possible value of
$$\frac{Y}{Z - X}$$

correct to three decimal places. (05 marks)

6. Two events are such that $P(A) = 0.7$, $P(B) = 0.2$ and $P(A/B) = 0.1$. Find:
- (a) $P(A \cup B)$. (03 marks)
- (b) $P(A \cap B')$. (02 marks)
7. A particle of weight 20 N is placed on a rough plane inclined at an angle of 40° to the horizontal. The coefficient of friction between the plane and the particle is $\frac{1}{4}$. When a horizontal force P is applied on the particle, it rests in equilibrium. Calculate the value of P . (05 marks)
8. A mobile phone dealer imports Nokia and Motorola phones. In a given consignment, 55% were Nokia and 45% were Motorola phones. The probability that a Nokia phone is defective is 4%. The probability that a Motorola phone is defective is 6%. A phone is picked at random from the consignment. Determine the probability that it is;
- (a) defective. (03 marks)
- (b) a Motorola given that it is defective. (02 marks)

SECTION B: (60 MARKS)

Answer any five questions from this section. All questions carry equal marks.

9. The table below shows marks obtained by 100 students in a Mathematics test.

Marks	Number of students
20 – < 40	5
40 – < 50	15
50 – < 55	10
55 – < 60	15
60 – < 70	25
70 – < 90	25
90 – < 100	5

- (a) Calculate the mean mark. (05 marks)
- (b) Draw a cumulative frequency curve (Ogive) and use it to find the;
- (i) median mark.
- (ii) range of the middle 40% of the marks. (07 marks)

10. Two bodies A and B of masses 3 kg and 2 kg respectively are 7 m apart on a smooth horizontal surface. A is moving directly towards B with a speed of 2 ms^{-1} and an acceleration of 0.3 ms^{-2} . B is moving in the same direction as A with a speed of 5 ms^{-1} and a retardation of 0.2 ms^{-2} . If the bodies collide and coalesce, calculate the;

- (a) time taken before collision occurs. (08 marks)
 (b) common velocity immediately after the collision. (04 marks)

11. (a) Use the trapezium rule with 6-ordinates to estimate

$$\int_{0.1}^{0.5} \frac{1}{2x+1} dx$$

correct to three significant figures. (06 marks)

(b) Evaluate $\int_{0.1}^{0.5} \frac{1}{2x+1} dx$

correct to three significant figures. (02 marks)

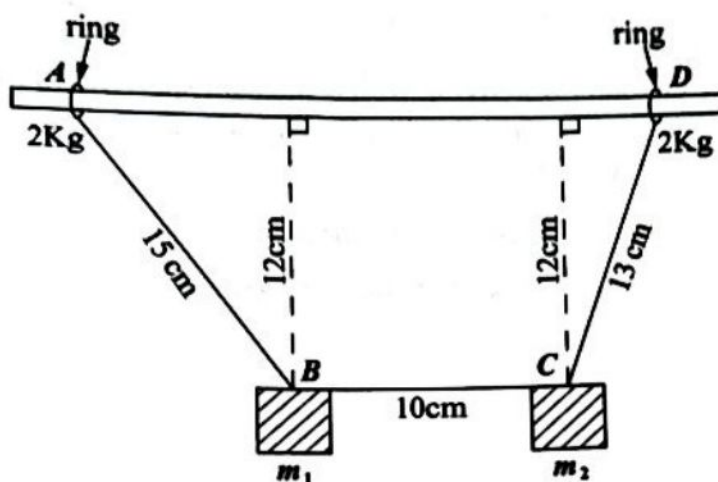
- (c) (i) Determine the percentage error in the estimation in (a) above, correct to two decimal places.
 (ii) Suggest how the percentage error may be reduced. (04 marks)

12. A continuous random variable X has a cumulative distribution function given by

$$F(x) = \begin{cases} 0; & x \leq 0 \\ \frac{k}{2} x^2; & 0 \leq x \leq 2 \\ k(6x - x^2 - 6); & 2 \leq x \leq 3 \\ 1; & x \geq 3 \end{cases}$$

- (a) Determine the value of k .
 Hence sketch the graph of $F(x)$. (08 marks)
 (b) Find the probability density function (pdf) of X . (04 marks)

13. The diagram below shows three strings $AB = 15$ cm, $BC = 10$ cm and $CD = 13$ cm. A and D are fixed to small rings each of mass 2 kg which can slide on a rough horizontal rail AD . Masses m_1 and m_2 are attached at B and C respectively. The system rests in equilibrium with BC at a distance of 12 cm below AD .



- (a) Show that $9m_1 = 5m_2$. (07 marks)
- (b) If the coefficient of friction between each ring and the rail is $\frac{1}{4}$ and the ring at A is on the point of slipping, determine the value of m_1 . (05 marks)
14. (a) (i) Draw on the same axes the graphs of $y = x \sin x$ and $y = e^x - 2$ for $0.5 \leq x \leq 1.5$.
(ii) Use your graphs to find an approximate root of the equation $2 - e^x + x \sin x = 0$. (06 marks)
- (b) Using Newton Raphson iterative formula and your approximate root in a (ii) above as the initial value, calculate the root of the given equation correct to three decimal places. (06 marks)
15. A certain football team has three matches to play. The probabilities of winning the first, second and third matches are $\frac{3}{5}$, $\frac{2}{5}$ and $\frac{1}{5}$ respectively.
- (a) Find the probability that the team wins;
(i) exactly two matches.
(ii) all matches.
(iii) no match. (07 marks)

Turn Over

- (b) If a random variable X is defined as "the number of matches won",
- (i) construct a probability distribution table for X .
 - (ii) calculate the expectation of X , $E(X)$. (05 marks)

16. Two airstrips P and Q are 100 km apart, P being west of Q . Two helicopters A and B fly simultaneously from P and Q respectively, at 11.00 a.m. Helicopter A is flying with a constant speed of 400 kmh^{-1} in a direction $\text{N}50^\circ\text{E}$. Helicopter B is flying at a constant speed of 500 kmh^{-1} in the direction $\text{N}70^\circ\text{W}$. Find the;
- (a) time when the helicopters are closest together. (08 marks)
 - (b) closest distance between the helicopters. (04 marks)

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① $r(t) = \begin{pmatrix} t^2 + 4t \\ 3t - t^3 \end{pmatrix} m$

$v(t) = \begin{pmatrix} 2t + 4 \\ 3 - 3t^2 \end{pmatrix} m s^{-1}$ M1 correct differentiation

$v(3) = \begin{pmatrix} 2(3) + 4 \\ 3 - 3(3^2) \end{pmatrix}$ M1 correct substitution for $t=3$

$= \begin{pmatrix} 10 \\ -24 \end{pmatrix}$ B1 correct output

Speed $= |v(3)| = \sqrt{10^2 + (-24)^2}$
 $= 26 m s^{-1}$

M1

A1

Obtaining the magnitude
if the 1st 2 M1 are scored
correct output with units
OK - 1

②	0.3	0.4	0.6
	3.00	$f(x)$	3.22

B1

beatron (mobile part) or (b1)
or implied

$\frac{f(x) - 3.00}{0.4 - 0.3} = \frac{3.22 - 3.00}{0.6 - 0.3}$

M1

Equating gradients correctly

$f(x) = 3.0733$

A1

correct output (at least 2 d.p.s)

(b)	0.9	x	1.2
	3.69	3.82	4.06

$\frac{x - 0.9}{3.82 - 3.69} = \frac{1.2 - 0.9}{4.06 - 3.69}$

M1

Equating gradients correctly

$x = 1.0054$

A1

correct output (at least 4 whole n.s.)

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$$(3) (a) \frac{P_{2008} \times 100}{P_{2006}} = 130 \quad - (1)$$

$$\frac{P_{2007} \times 100}{P_{2006}} = 112 \quad - (2)$$

B1 (for both expressions correct)

$$(1) - (2) \Rightarrow x = \frac{P_{2008} \times 100}{P_{2007}}$$

$$= \frac{130 \times 100}{112}$$

M1 making an attempt to find x correctly

$$= 116.0714$$

A1 correct output (at least 116)

$$(b) W.P.I = \frac{(105 \times 4) + (116.0714 \times 7) + (104 \times 2) + (113 \times 5)}{4 + 7 + 2 + 5}$$

M1 - correct attempt to find WPI

$$= 111.4167$$

A1 - correct output to at least 4 decimal places

M1 if A0 above is lost then A

$$(4) T = \frac{2\pi}{\omega} = \frac{2\pi}{3} \quad M1$$

(attempting to find omega(w)) or substitution in the correct formula or can be implied

$$\omega = 3 \text{ rad s}^{-1}$$

B1 (for 3 with or without units)

$$v^2 = \omega^2(A^2 - x^2)$$

$$3.6^2 = 3^2(A^2 - (A - 0.8)^2)$$

for correct substitution with or without units substituted as $3.6^2 = 3^2(A^2 - 0.8^2)$

$$1.44 = 1.6A - 0.64$$

$$2.08 = 1.6A$$

$$A = 1.3 \text{ m}$$

A1 correct output with or without units

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⑤ $e_x = 0.05$, $e_y = 0.005$, $e_z = 0.0005$ B1 B1 B1

$$\left(\frac{y}{z-x} \right)_{\max} = \frac{y_{\max}}{z_{\min} - x_{\max}}$$

$$= \frac{1.335}{2.2445 - 1.25}$$

$$= 1.34238$$

$$\approx 1.342$$

(for correct errors in e_x , e_y and e_z
(can be implied in the correct expression below)

M1 → correct substitution

A1 → correct output to 3 d.p.s

⑥ (a) $P(A/B) = \frac{P(A \cap B)}{P(B)}$

$$P(A \cap B) = 0.1 \times 0.2$$

$$= 0.02$$

B1 - attempt to find $P(A \cap B)$
($0.1 = \frac{P(A \cap B)}{0.2}$)

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= 0.7 + 0.2 - 0.02$$

$$= 0.88$$

M1 - correctly substituting in general $P(A \cup B)$ formula

A1 correct output

(b) $P(A \cap B') = P(A) - P(A \cap B)$

$$= 0.7 - 0.02$$

$$= 0.68$$

M1 - correct substitution

A1 - correct output

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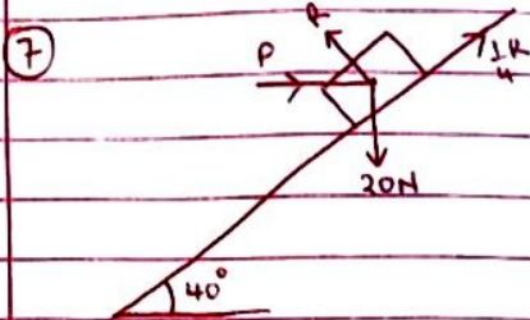
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B1 - for correct force diagram
 (4 forces seen correctly drawn w/ angles with θ or 40° and either $\frac{1}{4}R$

(i) $\frac{1}{4}R + P \cos 40^\circ = 20 \sin 40^\circ$ - (i) B1 - for correct resolution along the plane with or without θ and θ substituted
 (ii) $R = 20 \cos 40^\circ + P \sin 40^\circ$ - (ii) B1 - for correct resolution perpendicular to the plane

$\frac{1}{4}(20 \cos 40^\circ + P \sin 40^\circ) + P \cos 40^\circ = 20 \sin 40^\circ$ M1 - for combining/solving eqns simultaneously for P with θ and θ substituted
 $P = \frac{20(4 \sin 40^\circ - \cos 40^\circ)}{4 \cos 40^\circ + \sin 40^\circ}$
 $= 9.7390 \text{ N}$

A1 - for correct output with or without units to at least 1 dp

8) $P(N) = 0.55$, $P(M) = 0.45$, $P(D|N) = 0.04$, $P(D|M) = 0.06$

(a)

$P(D) = P(N \cap D) + P(M \cap D)$
 $= P(N) \cdot P(D|N) + P(M) \cdot P(D|M)$

for 2 correct arms of the probabilities seen

$\left(\frac{11 \times 1}{20 \times 1}\right) \left(\frac{3 \times 3}{20 \times 50}\right) = (0.55 \times 0.04) + (0.45 \times 0.06)$
 $= 0.049 \quad \text{OR} \quad \frac{49}{1000}$

B1 M1 - Addition of the two arms

A1 - Correct output to exactly 3 dps

(b) $P(M|D) = \frac{P(M \cap D)}{P(D)}$

if one arm is wrong give B0 then M1

$\left(\frac{3 \times 3}{50 \times 20}\right) = \frac{(0.06 \times 0.45)}{0.049}$
 $= 0.5510 \quad \text{OR} \quad \frac{27}{49}$

M1 - correct substitution in the correct formula

A1 - for correct output to at least 2 dps

NB give M1 if B1 M1 A0

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(9)

C.B	f	x	fx	C.f
20-40	5	30	150	5
40-50	15	45	675	20
50-55	10	52.5	525	30
55-60	15	57.5	862.5	45
60-70	25	65	1625	70
70-90	25	80	2000	95
90-100	5	95	475	100
Sum	100		6312.5	

All values of C.f values correct (can be implied in g.v)

(a) Mean = $\frac{\sum fx}{\sum f}$

BI correct sum of fx
BI BI BI All values of fx correct
All values of x correct

= $\frac{6312.5}{100}$

Accept correct
If BI for correct sum is missed then MI followed by A
Correct substitution in correct formula
(can be implied)

= 63.125 AI correct output (Accept 63.)

$63\frac{1}{8}$ or $\frac{505}{8}$ or

$63\frac{125}{1000}$ or $\frac{63125}{1000}$

(b) Median = 61.5

BI - for median value with location seen

Middle 40% = $P_{70} - P_{30}$

Accept median [61-65]

$P_{70} = 70$

$P_{30} = 55$

Range = $70 - 55$

= 15

MI - for finding the range with correct values with location seen or not seen

AI

for correct output

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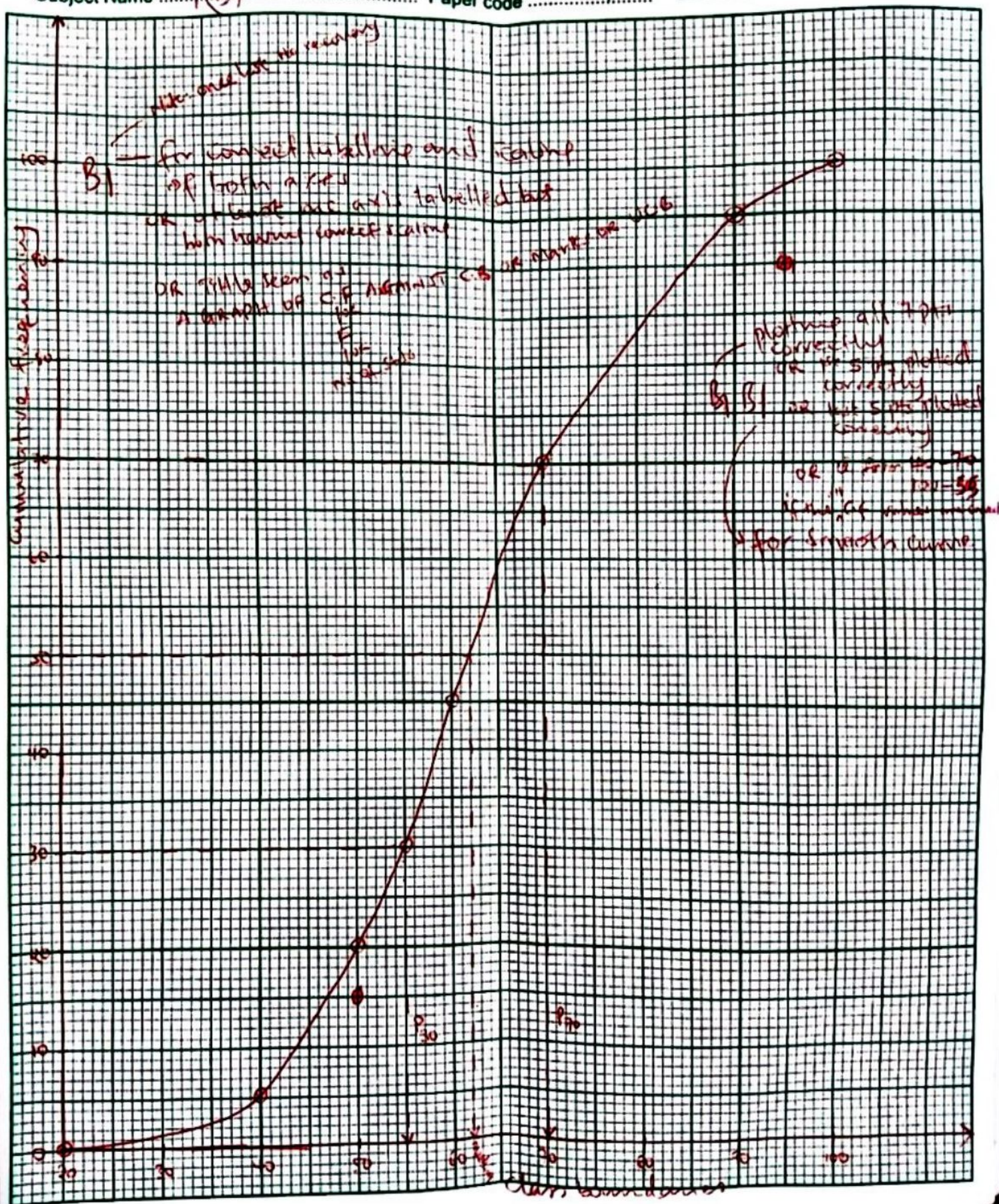
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(10)

(a)



$$a_A = 0.3 \text{ m/s}^2$$

$$a_B = -0.2 \text{ m/s}^2$$

B1 - for recognizing retardation
 $a = -0.2 \text{ m/s}^2$

$$u_A = 2 \text{ m/s}$$

$$u_B = 5 \text{ m/s}$$

$$s_A = (7+x) \text{ m}$$

$$s_B = x \text{ m}$$

B1 - for displacement of both
A and B
or x and $x-7$

$$\text{Using } s = ut + \frac{1}{2}at^2$$

$$\text{for A: } 7+x = 2t + \frac{1}{2} \times 0.3 \times t^2 \quad \text{--- (1)}$$

B1 - correct substitution in the
displacement formula
for A

$$\text{for B: } x = 5t - \frac{1}{2} \times 0.2 \times t^2 \quad \text{--- (2)}$$

Correct substitution in the
displacement formula for B

Substitute (2) in (1)

$$7+5t-0.1t^2 = 2t + 0.15t^2$$

M1 - for solving (1) and (2) simulta-
neously

$$0 = 0.25t^2 - 3t - 7 \text{ OR}$$

B1 - for coming out with correct
Quadratic equation

$$t^2 - 12t - 28 = 0$$

$$0.25t^2 - 3t - 7 = 0$$

$$t = \frac{3 \pm \sqrt{(-3)^2 - 4 \times 0.25 \times -7}}{2 \times 0.25}$$

M1 - for solving the quadratic
equation or can be implied
by embedding technology

$$t = 14, \text{ or } t = -2$$

$$\therefore t = 14 \text{ s}$$

A1

if for correct conclusion of $t = 14 \text{ s}$
with or without units

$$\text{NB } v_A = 2 + 0.3(14) \text{ A1 B1}$$

$$(b) v_A = 2 + 0.3(14) = 6.2 \text{ m/s}$$

B1 - for correct substitution with
correct output

$$v_B = 5 - 0.2 \times 14 = 2.2 \text{ m/s}$$

B1 - for correct substitution

$$m_A v_A + m_B v_B = (m_A + m_B) v_c$$

$$3(6.2) + 2(2.2) = (3+2) v_c$$

M1 - for correct substitution in the
momentum formula (even
equating)

$$v_c = 4.6 \text{ m/s}$$

correct output with or without
units.

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11(a) $h = \frac{0.5 - 0.1}{5}$

B1 - attempt to find h.

$= 0.08$

x	$f(x) = \frac{1}{2x+1}$	
0.1	0.83333	
0.18		0.73529
0.26		0.65789
0.34		0.59524
0.42		0.54348
0.50	0.50000	
Sum	1.33333	2.53190

B1
all values of x correct

for extreme end values
B1 B1 - for correct intermediate values of y to 4 d.p.s

M^y possible if the 1st 2 B's have been earned
i.e. B₁B₁B₁B₁ OR B₁B₁B₁B₁

$$\int_{0.1}^{0.5} \frac{1}{2x+1} dx \approx \frac{1}{2} \times 0.08 (1.33333 + 2(2.53190))$$

$$= 0.255885$$

$$\approx 0.256$$

for correct substitution
A1 - for correct output to 3 s.f.s

(b) $\int_{0.1}^{0.5} \frac{1}{2x+1} dx = \frac{1}{2} \ln(2x+1) \Big|_{0.1}^{0.5}$

M - for correct integration with limits

$$= \frac{1}{2} (\ln 2 - \ln(1.2))$$

$$= 0.255$$

A1 - for correct output to 3 s.f.s

(c) (i) error = $|0.255 - 0.256|$

$$= 0.001$$

B1 - for error with magnitude or implied (Accept if did not write 0.001)

percentage error = $\frac{0.001}{0.255} \times 100$

$$= 0.39$$

M - for correct substitution in % error formula

A1 - for correct output

(ii) By increasing the number of ordinates

B1 - earned when all the marks have been secured

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(a) $F(3) = 1$

31 — for equating $f(3) = 1$
(can be implied in the second step)

$K(6(3) - 3^2 - 6) = 1$

M1 — Attempt to find K by substituting $x = 3$

$K = \frac{1}{3}$

A1 — for correct output Accept 2dp or more

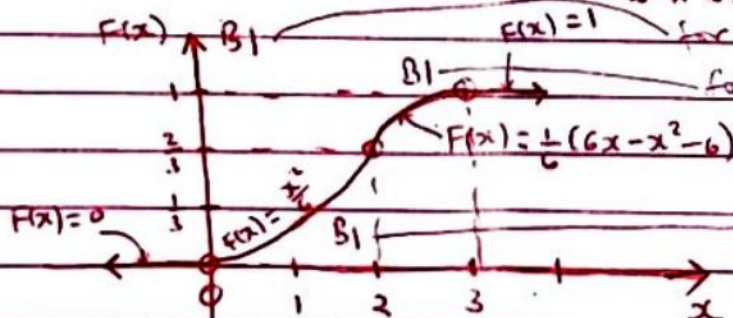
$$F(x) = \begin{cases} 0 & ; x < 0 \\ \frac{x^2}{6} & ; 0 \leq x \leq 2 \\ \frac{1}{3}(6x - x^2 - 6) & ; 2 \leq x \leq 3 \\ 1 & ; x > 3 \end{cases}$$

31 — for re writing $F(x)$ with K substituted

(can be implied in the graph)
if all the 4 regions are seen
with arrows for $x < 0$ & $x > 3$

x	0	2	3
F(x)	0	$\frac{2}{3}$	1

(Two tables in one) — can be implied in the graph
for the points of the 1st curve 0 to 2
for the points of the 2nd curve 2 to 3
with x substituted



for at least an axis labelled
for the curve $\frac{1}{3}(6x - x^2 - 6)$ drawn and labelled and the line for $F(x) = 1$ seen

for the curve $\frac{x^2}{6}$ drawn and labelled

Note: $F(x) = 1$ or $F(x) = 0$ may not be labelled.

Accept 1dp along $F(x)$ axis

NB: for as long as it's a curve
3. Accept if $F(x) = 0$ is not seen

(b) for $0 \leq x \leq 2$; $f(x) = \frac{d}{dx} \left(\frac{x^2}{6} \right) = \frac{x}{3}$

M1 — for the correct derivative with K substituted

for $2 \leq x \leq 3$; $f(x) = \frac{d}{dx} \left(\frac{1}{3}(6x - x^2 - 6) \right) = \frac{1}{3}(6 - 2x)$ OR $2(3 - x)$

M1 — for the correct derivative with K substituted

for $x > 3$; $f(x) = \frac{d}{dx} (1) = 0$

for $x < 0$; $f(x) = 0$

Hence, $f(x) = \begin{cases} \frac{1}{3}x & ; 0 \leq x \leq 2 \\ \frac{2}{3}(3 - x) & ; 2 \leq x \leq 3 \\ 0 & ; \text{otherwise} \end{cases}$

A1 — correct expression for the pdf

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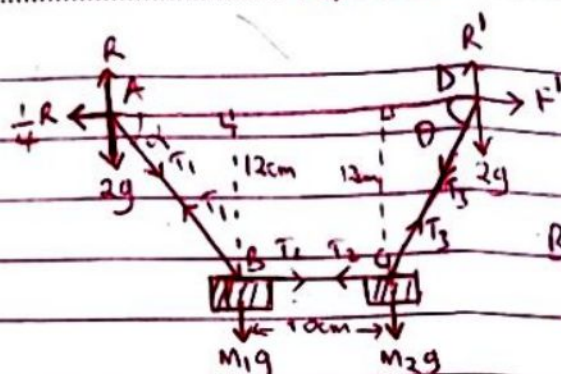
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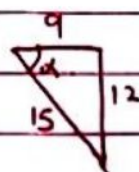
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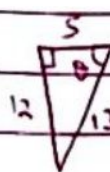


BI — for all the relevant force shown at A, B, C



$$\sin \alpha = \frac{12}{15} = \frac{4}{5}$$

$$\cos \alpha = \frac{9}{15} = \frac{3}{5}$$



$$\cos \theta = \frac{5}{13}$$

$$\sin \theta = \frac{12}{13}$$

BI — for at least ^{ing} sine ratio seen, $\alpha = 53.1^\circ$, $\theta =$

At B;

($\leftarrow$); $T_1 \cos \alpha = T_2$ — ① BI for HR resolution at B

($\uparrow$); $T_1 \sin \alpha = m_1 g$ — ② BI for VR resolution at B

② $\frac{12}{9} = \frac{m_1 g}{T_2}$

① $\frac{12}{9} = \frac{m_1 g}{T_2}$

$$m_1 g = \frac{12}{9} T_2$$

At C

($\leftarrow$); $T_2 = T_3 \cos \theta$ BI — for HR resolution

($\uparrow$); $m_2 g = T_3 \sin \theta$ BI — for VR resolution

$$\tan \theta = \frac{m_2 g}{T_2}$$

$$m_2 g = \frac{12}{5} T_2$$

$m_2 g = \frac{12}{5} \left(\frac{9}{12} m_1 g \right)$ M1 — for an attempt to combine the 4 equations

$9m_1 = 5m_2$ BI — for correct expression as required

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At A

$$\rightarrow; T_1 \cos \alpha = \frac{1}{4} R$$

B1. for correct HB resolution at A

$$T_1 \left(\frac{3}{5} \right) = \frac{1}{4} R$$

$$R = \frac{12}{5} T_1$$

$$\uparrow; R = T_1 \sin \alpha + 2g$$

B1. for correct VR resolution at A

$$R = T_1 \left(\frac{4}{5} \right) + 2g$$

$$\frac{12 T_1}{5} - \frac{4 T_1}{5} = 2g$$

M1. for combining the two resolutions above.

$$\frac{8 T_1}{5} = 2g \quad \text{but } T_1 = \frac{5 m_1 g}{4}$$

$$\frac{8}{5} \left(\frac{5 m_1 g}{4} \right) = 2g$$

$$\therefore m_1 = 1 \text{ kg} \quad \text{A1. for correct output with or without units}$$

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(a) $y = x \sin x$

x	0.5	0.75	1.0	1.25	1.50
y	0.2	0.5	0.8	1.2	1.5

B1 - at least 3 table values written to at least 1 dp within the range

$y = e^x - 2$

x	0.5	0.75	1.0	1.25	1.50
y	-0.4	0.1	0.7	1.5	2.50

B1 - at least 3 table values written to at least 1 dp

(a) Graph

(b) $f(x) = 2 - e^x + x \sin x$

$f'(x) = -e^x + 1.5 \sin x + x \cos x$

$x_{n+1} = x_n - \left(\frac{2 - e^{x_n} + x_n \sin x_n}{x_n \cos x_n + \sin x_n - e^{x_n}} \right)$

$x_0 = 1.1$

$x_1 = 1.1 - \left(\frac{2 - e^{1.1} + 1.1 \sin 1.1}{\sin 1.1 + 1.1 \cos 1.1 - e^{1.1}} \right)$

$= 1.0852$

$x_2 = 1.0852 - \left(\frac{2 - e^{1.0852} + 1.0852 \sin 1.0852}{\sin 1.0852 + 1.0852 \cos 1.0852 - e^{1.0852}} \right)$

$= 1.0850$

error = $|1.0850 - 1.0852|$

$= 0.0002$

$\therefore$ the root is 1.085

A1 - for the correct output written to exact 3 dps

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(To be fastened together with other answers to paper)

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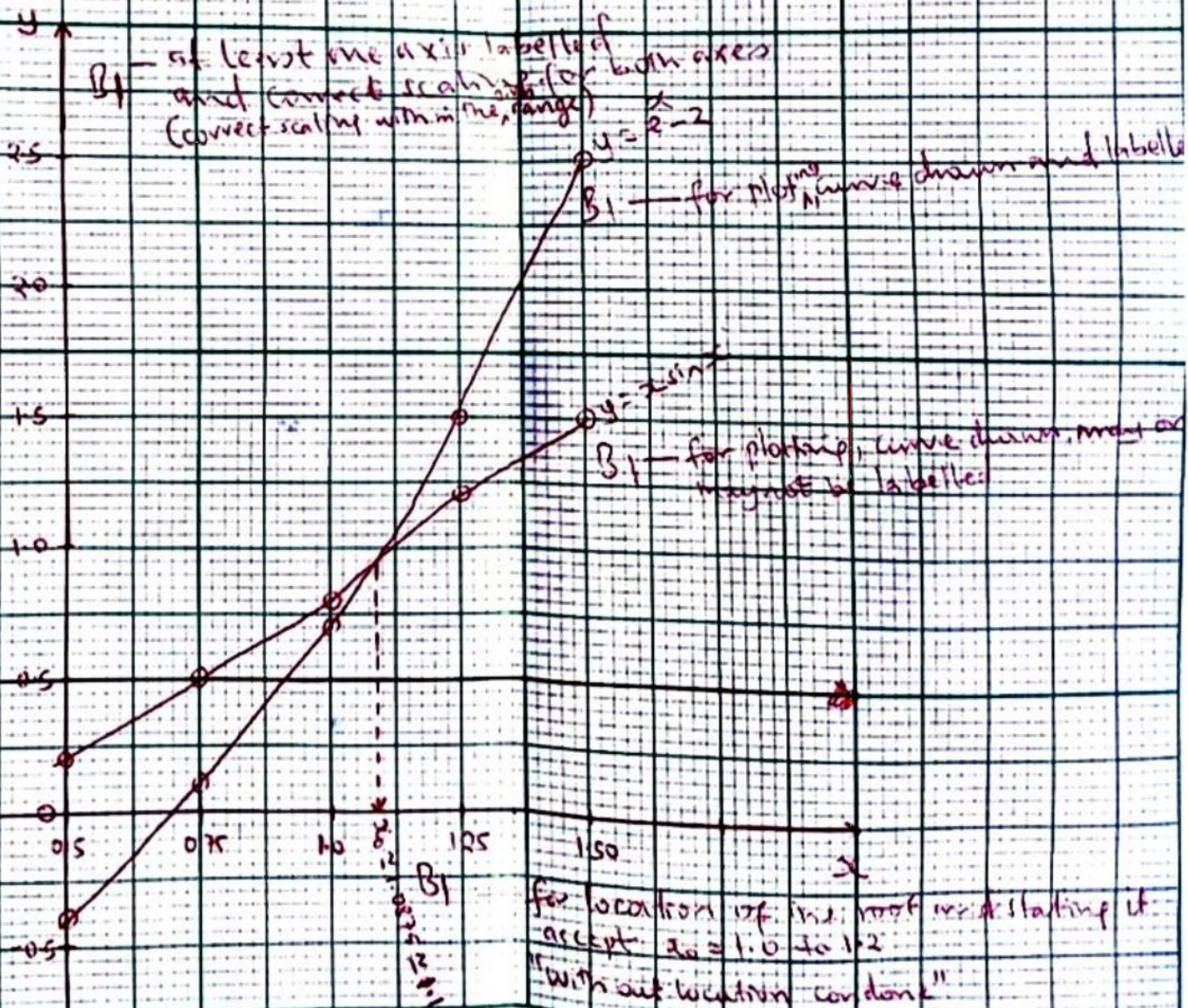
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(15) X = number of matches won.

a) (i) $P(X=2) = \left(\frac{3}{5} \cdot \frac{2}{5} \cdot \frac{4}{5}\right) + \left(\frac{1}{5} \cdot \frac{3}{5} \cdot \frac{1}{5}\right) + \left(\frac{2}{5} \cdot \frac{2}{5} \cdot \frac{1}{5}\right)$ for all arms correct
Adding arms

$= \frac{37}{125}$ OR 0.296

A1 - correct output to exactly 3 d.p.s

Note: M1 if any two arms are correct (B or M1 A1)

(ii) $P(X=3) = \left(\frac{3}{5} \cdot \frac{2}{5} \cdot \frac{1}{5}\right) = \frac{6}{125}$ OR 0.048

A1 - correct output to exactly 3 d.p.s
Correct multiplication of the probs

(iii) $P(X=0) = \left(\frac{2}{5} \cdot \frac{3}{5} \cdot \frac{4}{5}\right) = \frac{24}{125}$ OR 0.192

M1 A1 - correct output to exactly 3 d.p.s
for multiplication of probs

b) $P(X=1) = \left(\frac{3}{5} \cdot \frac{3}{5} \cdot \frac{4}{5}\right) + \left(\frac{2}{5} \cdot \frac{2}{5} \cdot \frac{4}{5}\right) + \left(\frac{2}{5} \cdot \frac{3}{5} \cdot \frac{1}{5}\right)$
 $= \frac{58}{125}$ OR 0.464

correct
for attempt to find $P(X=1)$
A1 - for correct output to exactly 3 d.p.s

Can be given in table on right

(i)

x	0	1	2	3
$P(X=x)$	$\frac{24}{125}$	$\frac{58}{125}$	$\frac{37}{125}$	$\frac{6}{125}$
$xP(X=x)$	0	$\frac{58}{125}$	$\frac{74}{125}$	$\frac{18}{125}$

and their corresponding
(for all $P(X=x)$ values)

B1

M1 - for product and sum

$\sum xP(X=x) = 1.2$ A1 - correct output

(ii) $E(X) = 1.2$

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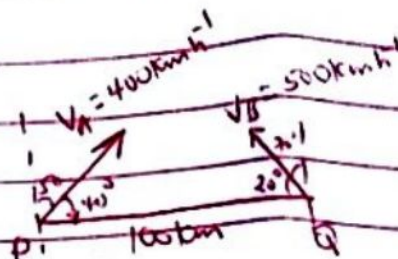
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16 (a)



let P be the origin

$$\vec{r}_A(0) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad \vec{r}_B(0) = \begin{pmatrix} 100 \\ 0 \end{pmatrix}$$

$$\vec{v}_A = 400 \begin{pmatrix} \cos 40^\circ \\ \sin 40^\circ \end{pmatrix} \quad \text{B1 - for velocity of A expressed in vector form}$$

$$= \begin{pmatrix} 306.4178 \\ 257.1150 \end{pmatrix} \quad \text{accept upto at least 2 d.p.}$$

$$\vec{v}_B = 500 \begin{pmatrix} -\cos 20^\circ \\ \sin 20^\circ \end{pmatrix} \quad \text{B1 - for velocity of B expressed in vector form}$$

$$\vec{v}_B = \begin{pmatrix} -469.8463 \\ 171.0101 \end{pmatrix} \quad \text{accept upto at least 2 d.p.}$$

$$\vec{v}_{AB} = \begin{pmatrix} 306.4178 \\ 257.1150 \end{pmatrix} - \begin{pmatrix} -469.8463 \\ 171.0101 \end{pmatrix} \quad \text{B1 - for attempt to find relative velocity}$$

$$= \begin{pmatrix} 776.2641 \\ 86.1049 \end{pmatrix}$$

$$\vec{r}_A(t) = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 306.4178 \\ 257.1150 \end{pmatrix} t \quad \text{B1 - for relative position of A at any time } t$$

$$= \begin{pmatrix} 306.4178t \\ 257.1150t \end{pmatrix}$$

$$\vec{r}_B(t) = \begin{pmatrix} 100 \\ 0 \end{pmatrix} + \begin{pmatrix} -469.8463 \\ 171.0101 \end{pmatrix} t \quad \text{B1 - for relative position of B at any time } t$$

$$= \begin{pmatrix} 100 - 469.8463t \\ 171.0101t \end{pmatrix}$$

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$$\vec{A\hat{r}_B}(t) = \begin{pmatrix} 306.4178t \\ 257.1150t \end{pmatrix} - \begin{pmatrix} 100 - 469.8463t \\ 171.0101t \end{pmatrix} \quad B1$$

$$= \begin{pmatrix} 776.2641t - 100 \\ 86.1049t \end{pmatrix}$$

an attempt to find
for a relative
position of A to B
or position of A relative
to B.

for closest distance,

$$A\hat{V}_B \cdot A\hat{r}_B(t) = 0$$

$$\begin{pmatrix} 776.2641 \\ 86.1049 \end{pmatrix} \cdot \begin{pmatrix} 776.2641t - 100 \\ 86.1049t \end{pmatrix} = 0 \quad M1$$

for dotting
relative velocity
and relative
position and
equating to zero

$$602585.9529t - 77626.41 + 7414.0538t = 0$$

$$610000.0067t = 77626.41$$

$$t = 0.1273 \text{ hrs} \quad B1 \quad \text{--- for the output to 2 d.p.}$$

$$= 0.1273 \times 60$$

$$= 7.638 \text{ min.}$$

$$\approx 8 \text{ minutes}$$

Time is 11:08 am

A1 --- Correct time with or without
a.m.

$$(b) \vec{A\hat{r}_B}(0.1273) = \begin{pmatrix} 776.2641 \times 0.1273 - 100 \\ 86.1049 \times 0.1273 \end{pmatrix} \quad B1 \quad \text{--- for substitution of } t$$

$$= \begin{pmatrix} -1.1816 \\ 10.9612 \end{pmatrix}$$

$$\text{Shortest distance} = |\vec{A\hat{r}_B}(t)|$$

$$= \sqrt{(-1.1816)^2 + 10.9612^2}$$

M1 --- for attempt
to find the
distance

$$= 11.0247 \text{ km}$$

A1 --- Correct output
to at least a whole
number with or without
the unit.